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Efficient Union Contracts in the Presence of Homogeneous Labor and Differentiated Unions

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This paper discusses the features of the labor market outcome in the presence of homogeneous labor and multiple unions. It is argued that contract curve agreements, or at least efficient bargaining among unions, may be improved upon by a solution with differential or non-uniform wage payments for workers affiliated to different unions.

The equilibrium solution in terms of employment and/or wage bill shares implied for each union with uniform and multiple wage is investigated and confronted for the cases of:
a. efficient bargaining among the unions
b. fully efficient bargaining,
considering Stone-Geary union utility functions and labor demand is linear.
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1. INTRODUCTION

This paper enlarges the study of the labor market outcomes in the presence of multiple unions, considering the possibility of different (net) wage payments to be allocated to homogeneous(ly productive) workers affiliated to different unions. Product market price-discrimination is not an adequate analog to the scenario invoked below (neither will "right-to-manage" be the motive behind the implicit maximands); rather, (it is as if) there will be transfers of employment and wage payments among unions but the unit of labor will ultimately be sold at a unique (average) price (wage).

The multiple union solution has been studied previously in the literature. Oswald (1979) [1] departs from unions with price competition strategies - framework also used by Gylfason and Lindbeck (1984a and 1984b)[2] - and derives the properties of the Cournot-Nash equilibrium, comparing it with the Stackelberg one and even describing efficient cooperation between unions. He assumes heterogeneous labor with substitutability among workers. In some sense, Martins and Coimbra (1997b) use the same technique; only, with perfect substitutability – as considered by Hart (1982) for syndicates in a particular market, we use quantity instead of price competition. There, we concluded that unions where the workers were perfect substitutes in production had advantage in coordinating their bargaining.

This paper highlights the fact that the model with efficient bargaining among many unions with uniform wages imposed in previous research suggested by the fact that labor is homogeneous in the context is in fact a second best solution. It is possible to improve it allowing transfers across union members, or non-uniform wage payments. Such rearrangement can be processed internally within the unions coalition; or work through employers behavior (being responsible the redistribution process) which pay differently to workers according to unions affiliation even if they are perfect substitutes in production. In real life, the latter could transmit to the labor market analyst the impression of imperfect substitutability between workers in production when in fact there is not.

The sorting of wage payments in such a way that higher unemployment risk is associated with a wage premium is well documented in the hedonic wage literature. The possibility of higher total compensation for a less stable employment contract (i.e., with lower employment probability) would, thus, be replicated in this research. The situation arises here due to different individual/union's preferences over the probability of (un)employment-

[1] Citing Rosen (1970) as the first author to recognize strategic interdependency among unions.

[2] Also Davidson (1988), Dixon (1988), Dowrick (1989), Jun (1989) and Dobson (1994), for example, where the effect of the existence of oligopoly in the product market is investigated.

wage mix [3] and not due to intrinsic worker productivity differences. On average, employers would pay the marginal product of labor for this input, even if differently to each type or union's affiliates.

The implementation of this type of solutions may be difficult it may be difficult for unions to justify to members the payment of, after all, different wages to workers with the same productivity. Also, employers sorting contracts according to unions relative preferences over the wage (probability of) employment mix may be difficult to implement. The efficient bargaining outcome with uniform wage payments may therefore be a feasible second best solution compared to this equilibrium.

In section 2, we outline some results connected with efficient bargaining among unions and among unions and employers for homogeneous labor and uniform wage setting. In section 3, transfers are considered for those two scenarios. We use Stone-Geary union utility preferences and a linear demand to derive more specific conclusions and comparison of the four scenarios in section IV. The exposition ends with a brief summary in section V.

2. STANDARD EFFICIENT BARGAINING SOLUTIONS

Assume that there are n unions in the economy. The unions maximize the general utility function $U^i(L_i, W)$, increasing in the arguments employment, L_i, and W, wage and quasi-concave, for which U^i_L/U^i_W the marginal rate of substitution between employment and wage decrease with L_i and increases with W. Employment contracts are under closed-shop agreements, i.e., the firm(s) can only hire unionized workers. Demand is of the form:

$$\sum_{i=1}^{n} L_i = L(W) \tag{1}$$

or its inverse:

$$W = W\left(\sum_{i=1}^{n} L_i\right) \tag{2}$$

being negatively sloped, coming from maximization (in $L = \sum_{i=1}^{n} L_i$) of the (aggregate) profit function $\prod(L, W) = PF(\sum_{i=1}^{n} L_i) - W\sum_{i=1}^{n} L_i$. Therefore, (2) establishes the value of the marginal product of labor, equal for all types of workers [4].

[3] Oswald (1979) suggests such a link between unions' utility functions and members preferences.

[4] Nevertheless, most of the results below would also apply if this function represented the marginal revenue product of labor and if firms did not behave competitively in the product market.

2.1. Efficient Cooperation among Unions

Assume the unions cooperate with each other and we can extend the Nash-maximand approach to model n-union cooperative behavior. Then, unions maximize:

$$\max_{L_1,L_2,\ldots,L_n} \prod_{i=1}^{n} \left\{ U^i \left[L_i, W\left(\sum_{j=1}^{n} L_j\right)\right] - \overline{U}^i \right\}^{\delta_i} \tag{3}$$

δ_i is related to the strength of union i within the coalition — as justified by Svejnar (1986), extending the Nash-Zeuthen-Harsanyi solution. δ_i/δ_j can be associated in "fair gambles" with M_i/M_j, where M_i denotes number of members of union i [5]. Eventually, (3) could represent the utility function of a unique union with workers with different preferences over the wage-employment mix, having n types of workers, with M_i workers of type i, $i = 1, 2, \ldots, n$ — that, is an extended monopoly union maximand. With perfect substitution between workers, ultimately the wage paid by the firm(s) must be the same for all workers.

F.O.C yield:

$$\delta_i U_L^i/[U^i(L_i, W) - \overline{U}^i] = -W_L \sum_{j=1}^{n} \delta_j U_W^j/[U^j(L_j, W) - \overline{U}^j] \qquad i = 1, 2, \ldots, n \tag{4}$$

Unions for which δ_i is higher — unions that are stronger within the coalition will have higher values of $[U^i(L_i, W) - \overline{U}^i]/U_L^i$; given that W is equal for all unions, provided that utility functions are similar, this would suggest larger L_i. As the right-hand-side of (4) is the same for any i:

$$\delta_i U_L^i/[U^i(L_i, W) - \overline{U}^i] = \delta_j U_L^j/[U^j(L_j, W) - \overline{U}^j] \tag{5}$$

This can be seen as a distribution (across unions) equation.

The equilibrium will obey labor demand and [6]:

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[5] If we consider that $\sum_{i=1}^{n} \delta_i = 1$, then we can link $\delta_i = M_i / \sum_{j=1}^{n} M_j$. See Martins and Coimbra (1997a) for a justification of the relation between δ_i and number of members of union i.

[6] Efficiency conditions, in Edgeworth tradition, would also come from the solution of the problem

$$\begin{aligned} & \max_{L_1,L_2,\ldots,L_n,W} U^1(L_1, W) \\ s.t.: \quad & U^j(L_j, W) \geq \overline{U}^j, \quad j = 2, 3, \ldots, n \\ & W = W(\sum_{i=1}^{n} L_i) \end{aligned}$$

$$\sum_{i=1}^{n} U_W^i / U_L^i = -L_W = -1/W_L \tag{6}$$

The sum of the Marginal Rates of Substitution between wage and employment of the several unions equal the labor demand slope [7]. This case reproduces the (an aggregate) monopoly union behavior.

2.2. Fully Efficient Bargaining

$$\max_{L_1,\ldots,L_n,W} \prod_{i=1}^{n} [U^i(L_i, W) - \overline{U}^i]^{\delta_i} [\Pi[(\sum_{i=1}^{n} L_i), W] - \overline{\Pi}] \tag{7}$$

δ_i represents the strength of union i relative to the employer side. And δ_i/δ_j will represent the strength of union i relative to union j. This model structure compares to the previous as, in the one-union case, efficient bargaining in McDonald and Solow's (1981) sense compares to the monopoly union problem. A bargaining with equal strength between unions (together) and employers will require:

$$\sum_{i=1}^{n} \delta_i = 1 \tag{8}$$

F.O.C. will yield (5) and also [8]:

$$\sum_{i=1}^{n} U_W^i / U_L^i = \Pi_W / \Pi_L = -\sum_{i=1}^{n} L_i / [PF_L - W] \tag{9}$$

(9) reproduces the tangency condition of efficient bargaining contract curves: the sum of the Marginal Rates of Substitution between wage and employment of the several unions equal the slope of the aggregate isoprofit curve. The comparison of the two forms — (6) and (9) — for a union duopoly can be found in Martins and Coimbra (1997a), and for an oligopoly in Martins and Coimbra (1997b); without a particular form for

[7] See Martins and Coimbra (1997a) for an interpretation of the result.

[8] These properties would also arise from the solution of the problem

$$\begin{aligned} &\max_{L_1,L_2,\ldots,L_n,W} U^1(L_1, W) \\ s.t.: \quad & U^j(L_j, W) \geq \overline{U}^j, \quad j = 2, 3, \ldots, n \\ & \Pi(\sum_{i=1}^{n} L_i, W) \geq \overline{\Pi} \end{aligned}$$

the union utility function and labor demand not much can be add. The main important feature of the solutions is that efficient cooperation with the employer side leads to an equilibrium where wages are higher than the marginal product of labor as it occurs in the traditional contract curve solution when there is only one union.

3. EFFICIENT BARGAINING AND TRANSFERS AMONG UNIONS

Consider that it is possible to operate transfers among union members, in such a way that members of different unions, even if identical with respect to productivity potential, receive different wage payments. Hence, in this section, we outline and confront the features of the general equilibrium solution in a multiple wage system for the cases of (only) cooperation among unions and full cooperation among unions and with the employer side. Also, comparisons with the uniform wage scenario — the second best solution — are also derived. Notice that conclusions and comparisons are drawn for fixed δ_i's — i.e., we compare "first" and "second" best solutions for given relative strength of unions in the coalition.

3.1. Efficient Bargaining among Unions

Assume, then, that union's i worker when "fully" employed receives a wage:

$$W_i = W + t_i = PF_L + t_i \tag{10}$$

being t_i positive or negative in such a way that:

$$\sum_{i=1}^{n} t_i L_i = 0 \tag{11}$$

Then, efficient bargaining among unions solves:

$$\max_{L_1,L_2,\ldots,L_n,t_1,t_2,\ldots,t_n} \prod_{i=1}^{n} \left\{ U^i \left[L_i, W(\sum_{j=1}^{n} L_j) + t_i \right] - \overline{U}^i \right\}^{\delta_i} \tag{12}$$

$$\text{s.t.}: \sum_{i=1}^{n} t_i L_i = 0$$

This problem will obviously lead to a higher maximand than the case of efficient bargaining among the unions i.e., (3). However, it may lead to a worse outcome for the unions than fully efficient bargaining i.e., (7), because for the same level of global employment, L, the average wage in (12) is still restricted to obey demand. The gain of (12) relative to (3) comes from a better redistribution of employment and worker compensation in order to take into account the different relative preferences for the mix across the unions. When compared with (7), we have this gain in (12), but it can be outweighed by the (efficiency) gain in (7) or loss in (12) from the efficient negotiation with the employer side which allows all workers to be paid more than the marginal product of labor.

The extraction of analytical conclusions turned out to be not so straightforward as in the cases of the previous section. We therefore present below the steps of some tedious algebra that allowed us to arrive to some results.

Constructing the lagrangean of problem (12), denoting by Π the full maximand, F.O.C. yield:

$$\delta_i \Pi U_L^i/[U^i(L_i, W+t_i) - \overline{U}^i] + W_L \sum_{j=1}^{n} \delta_j \Pi U_W^j/[U^j(L_j, W+t_j) - \overline{U}^j] + \lambda t_i = 0 \quad (13)$$

$$i = 1, 2, \ldots, n$$

and

$$\delta_i \Pi U_W^i/[U^i(L_i, W+t_i) - \overline{U}^i] + \lambda L_i = 0, \quad i = 1, 2, \ldots, n \qquad (14)$$

Replacing in (13):

$$\{\delta_i(U_L^i - U_W^i t_i/L_i)/[U^i(L_i, W+t_i) - \overline{U}^i]\} = -W_L \sum_{j=1}^{n} \delta_j U_W^j/[U^j(L_j, W+t_j) - \overline{U}^j] \qquad (15)$$

This expression differs from (4) in general, the left-hand side of (15), evaluated at $(W+t_i, L_i)$ and including the restriction on transfers will differ from the left-hand side of (4) evaluated at (W, L_i). At first glance, for fixed values of the right hand-side, we would require that unions for which t_i is higher would exhibit higher U_L^i or U_L^i/U_W^i in general, implying a lower L_i than in the no transfers case.

From (15):

$$\delta_i(U_L^i - U_W^i t_i/L_i)/[U^i(L_i, W+t_i) - \overline{U}^i] = \delta_j(U_L^j - U_W^j t_j/L_j)/[U^j(L_j, W+t_j) - \overline{U}^j] \qquad (16)$$

This determines distribution, having an analog with (5).

Let us consider again (14) and replace (13) in the following way:

$$\delta_i \Pi U_L^i/[U^i(L_i, W+t_i) - \overline{U}^i] - \lambda(W_L \sum_{j=1}^{n} L_j - t_i) = 0 \quad i = 1, 2, \ldots, n \quad (17)$$

Then, dividing after passing the terms in (17) for the right-hand side, by (14) we get:

$$L_i U_L^i / U_W^i = -W_L \sum_{j=1}^{n} L_j + t_i, \quad i = 1, 2, \ldots, n \tag{18}$$

Notice that (18) implies that:

$$L_i U_L^i / U_W^i - t_i = L_j U_L^j / U_W^j - t_j = -W_L L \tag{19}$$

Replacing in (16):

$$\delta_i (U_W^i / L_i) / [U^i(L_i, W + t_i) - \overline{U}^i] = \delta_j (U_W^j / L_j) / [U^j(L_j, W + t_j) - \overline{U}^j] \tag{20}$$

Multiplying both sides of (18) by L_i and summing over i we get:

$$\sum_{i=1}^{n} L_i^2 U_L^i / U_W^i = -W_L L^2 \tag{21}$$

or, being $s_i = L_i / L$ the employment share of union i:

$$\sum_{i=1}^{n} s_i^2 U_L^i / U_W^i = -W_L \tag{22}$$

or:

$$\sum_{i=1}^{n} s_i (L_i / W) U_L^i / U_W^i = 1/\eta \tag{23}$$

or yet, denoting the share of the total wage bill that will go to union i, $\omega_i = W_i L_i / (WL)$,

$$\sum_{i=1}^{n} \omega_i (-d \ln W_i / d \ln L_i |_{U=\overline{U}}) = \sum_{i=1}^{n} \omega_i E_i = 1/\eta \tag{24}$$

where $E_i = -d \ln W_i / d \ln L_i |_{U=\overline{U}} = U_L^i L_i / (U_W^i W_i)$ represents a similar concept to the logarithmic marginal rate of substitution between wage and employment along the union's indifference curve [9] (in a particular point of the indifference curve of union i, the union is willing to exchange a 1% increase in employment for an E_i% decrease in wages; the higher E, the stronger the union's preference for employment.).

[9] See, for example, Pencavel (1984) or Clark and Oswald (1993) for a definition.

We could also derive:

$$\sum_{i=1}^{n} s_i U_L^i / U_W^i - \sum_{i=1}^{n} t_i / L = -W_L n \tag{25}$$

If $\sum_{i=1}^{n} t_i = 0$, or $\sum_{i=1}^{n} t_i / L$ is negligible:

$$n / (\sum_{i=1}^{n} s_i U_L^i / U_W^i) = -L_W \tag{26}$$

(24) states that n times the weighted (by the employment share) harmonic mean of the marginal rates of substitution between wage and employment, U_W^i / U_L^i, must equal the (absolute value of the) labor demand slope.

All conditions are very different from (6). However, (24) offers some analogy in the left hand-side: it represents the sum over all unions of the (weighted) harmonic mean of U_W^i / U_L^i; in (6) we simply had the sum of these n Marginal Rates of Substitution — or n times its arithmetic (unweighted) mean. The reason for the different treatment is allowing (net) wages to vary in the problem from which (24) was derived. Recall that the harmonic mean is smaller than the arithmetic mean (when the same weights are used); for equality of the left hand-side of (6) and (24) to hold, in the presence of transfers, unions with higher U_L^i / U_W^i — that require a higher wage improvement to give up one unit of employment to stay in the same utility level — (than average) should have low s_i's (lower than $1/n$) and vice-versa.

The same type of considerations could be deducted if we rearrange (6) of uniform wages as yielding $1/\sum_{i=1}^{n} s_i/E_i = 1/\sum_{i=1}^{n} \omega_i/E_i = 1/\eta$ and compared it with (23). Unions with higher E_i — higher preference for employment must now be allocated lower wage bill share ω_i than with uniform wages.

PROPOSITION 1. *Possibility of transfers across union members in the presence of efficient bargaining (cooperation) between the* n *unions — union collusion — will originate a redistribution gain relative to the case where worker compensation is forced to be uniform. The optimal allocation suggests that:*

1.1. Unions where the workers receive a higher wage premium should exhibit lower employment of its members relative to the case where transfers were not possible.

1.2. Unions with higher marginal rate of substitution between wage and employment — that require higher wage compensation in exchange for one *unit of labor to stay in the same indifference curve — will be allocated lower employment.*

1.3. Unions with higher logarithmic marginal rate of substitution between wage and employment — that require higher wage compensation in exchange for one percent decrease in employment to stay in the same indifference curve — will be allocated a lower wage bill share than with uniform wages.

3.2. Fully Efficient Bargaining

With globally efficient bargaining it is reasonable to conceive that wages negotiated by employers with different unions these with different bargaining strength and different preferences over the employment-wage mix would be different. Let W_i denote wage paid to workers of union i. Then, the profit function is

$$\Pi(L_1, L_2, \ldots, L_n, W_1, W_2, \ldots, W_n) = PF(\sum_{i=1}^{n} L_i) - \sum_{i=1}^{n} W_i L_i \qquad (27)$$

The problem becomes:

$$\max_{L_1,\ldots,L_n,W_1,\ldots,W_n} \prod_{i=1}^{n} [U^i(L_i, W_i) - \overline{U}^i]^{\delta_i} [\Pi(L_1, \ldots, L_n, W_1, \ldots, W_n) - \overline{\Pi}] \qquad (28)$$

In this case, the distribution equation becomes:

$$\delta_i (U_L^i / \Pi_{L_i}) / [U^i(L_i, W_i) - \overline{U}^i] = \delta_j (U_L^j / \Pi_{L_j}) / [U^j(L_j, W_j) - \overline{U}^j] \qquad (29)$$

or

$$\delta_i (U_W^i / \Pi_{W_i}) / [U^i(L_i, W_i) - \overline{U}^i] = \delta_j (U_W^j / \Pi_{W_j}) / [U^j(L_j, W_j) - \overline{U}^j] \qquad (30)$$

which corresponds to

$$\delta_i (U_W^i / L_i) / [U^i(L_i, W_i) - \overline{U}^i] = \delta_j (U_W^j / L_j) / [U^j(L_j, W_i) - \overline{U}^j] \qquad (31)$$

Interestingly, (29) is equal to condition (20), i.e., distribution conditions among unions in the coalition end up to be similar with and without cooperation with the employer side.

An efficient allocation obeys:

$$U_W^i / U_L^i = \Pi_{W_i} / \Pi_{L_i} = -L_i / [PF_L - W_i] \qquad (32)$$

From (30) we also see that higher marginal rate of substitution U^i_W/U^i_L will originate a higher level of employment, L_i, relative to wage, W_i, for union i (once PF_L is fixed for all i). All types of workers will be paid higher than the marginal product of labor.

Summing (30) on i, we still have a different efficiency condition from (9). Let us compare it also with (15).

Consider that we define the average wage paid by employers as:

$$W = \sum_{i=1}^{n} W_i L_i / \sum_{i=1}^{n} L_i \tag{33}$$

This wage has correspondence with W of the previous case. Manipulating (30):

$$W_i - PF_L = L_i U^i_L / U^i_W \tag{34}$$

If we multiply by L_i and sum over i, we get:

$$\sum_{i=1}^{n} W_i L_i - PF_L L = \sum_{i=1}^{n} L_i^2 U^i_L / U^i_W \tag{35}$$

Dividing by $L^2 = (\sum_{i=1}^{n} L_i)^2$, we have:

$$(W - PF_L)/L = \sum_{i=1}^{n} s_i^2 U^i_L / U^i_W \tag{36}$$

(34) reproduces (22) if we replace $-W_L$ by $(W - PF_L)/L$, the latter corresponding to Π_L/Π_W of the uniform wage profit functions. The expression could be rearranged to present similar comments as (24) yielded.

In terms of the wage mark-up and the analog of (23):

$$(W - PF_L)/W = \sum_{i=1}^{n} \omega_i E_i \tag{37}$$

PROPOSITION 2. *If unions are allowed to set different wages and we consider fully efficient bargaining:*

2.1. They may pick different combinations of wages and employment than in the previous cases.

2.2. All types of workers will be now paid above the marginal product of labor.

2.3. Distribution among unions may show similar patterns to the case of no cooperation with the employer side. However, it will be different from the cases where uniform wages are imposed.

2.4.Aggregate conditions suggest that the relation between only efficient bargaining among unions and full efficient bargaining is similar for the cases of uniform and multiple wage payments.

4. AN ANALYTICAL EXAMPLE

Assume that the unions maximize the special case of the Stone-Geary utility function:

$$U^i(L_i, W) = W^{\theta_i} L_i^{(1-\theta_i)} \tag{38}$$

$\gamma_i = (1-\theta_i)/\theta_i$ represents union i's relative (to wage) preference for employment and equals the concept of E_i, the logarithmic marginal rate of substitution between wage and employment invoked above, constant for this function. When necessary, we use a linear demand schedule:

$$W = a - b(\sum_{i=1}^{n} L_i) \tag{39}$$

We denote by $\overline{\theta}$:

$$\overline{\theta} = \sum_{i=1}^{n} \delta_i \theta_i / \sum_{i=1}^{n} \delta_i \tag{40}$$

i.e., $\overline{\theta}$ is the weighted average of the θ_i's, the weights being the strength parameters in the coalition.

The results for oligopoly, efficient bargaining and fully efficient bargaining were derived in Martins and Coimbra (1997b) for uniform wage payments and we only reproduce here the relevant results for employment and wage bill share comparison.

4.1. Efficient Bargaining without Transfers

Assume, as usual, that $\overline{U}^i = 0, i = 1, 2, \ldots, n$. In this setting, (5) yields:

$$s_i = \delta_i(1-\theta_i)/\sum_{j=1}^{n} \delta_j(1-\theta_j) = (\delta_i/\sum_{j=1}^{n} \delta_j)(1-\theta_i)/(1-\overline{\theta}) \tag{41}$$

where s_i is the employment share of union i, i.e., $s_i = L_i/L = L_i/\sum_{j=1}^{n} L_j$. (9) yields:

$$\sum_{i=1}^{n} [\theta_i/(1-\theta_i)] s_i = \eta \tag{42}$$

and:

$$\begin{aligned}
\eta &= \sum_{i=1}^{n} \delta_i\theta_i / \sum_{i=1}^{n} \delta_i(1-\theta_i) \\
&= [\sum_{i=1}^{n} \delta_i\theta_i / \sum_{i=1}^{n} \delta_i]/\{[1-[\sum_{i=1}^{n} \delta_i\theta_i / \sum_{i=1}^{n} \delta_i] \\
&= \overline{\theta}/(1-\overline{\theta})
\end{aligned} \tag{43}$$

If demand is linear, one can show that:

$$L_i = (a/b)\delta_i(1-\theta_i)/\sum_{i=1}^{n} \delta_i, \quad i = 1, 2, \ldots, n \tag{44}$$

$$L = (a/b)\sum_{i=1}^{n}[(1-\theta_i)\delta_i]/\sum_{i=1}^{n} \delta_i = (a/b)(1-\overline{\theta}) \tag{45}$$

and

$$W = a\sum_{i=1}^{n} \theta_i\delta_i / \sum_{i=1}^{n} \delta_i = a\overline{\theta} \tag{46}$$

The aggregate outcome seems independent from number of unions and reproduces the monopoly union result [10].

4.2. Fully Efficient Bargaining without Transfers

With fully efficient bargaining, (39) holds and employment and wage bill shares will be the same as with only efficient contracts among unions. One can show that

$$W = (a - bL)\overline{\theta}/(2\overline{\theta} - 1) \tag{47}$$

The aggregate contract curve seems independent from the number of unions.

If $\overline{\theta} > 0.5$, for positive wage, the marginal product of labor will be positive. If $\overline{\theta} < 0.5$, aggregate employment will be pushed till a point where the marginal product of labor is negative. For given aggregate employment, contract curve agreements will imply a larger wage level the larger is the average union preferences for wage, $\overline{\theta}$[11].

4.3. Efficient Bargaining with Transfers among Unions

[10] See Martins and Coimbra (1997b) for additional comments.
[11] See Martins and Coimbra (1997b) for additional comments.

Consider our Stone-Geary utility functions. Then (24) yields:

$$\sum_{i=1}^{n} s_i(1-\theta_i)/\theta_i + \sum_{i=1}^{n} s_i(t_i/W)(1-\theta_i)/\theta_i = 1/\eta \tag{48}$$

and (22)

$$\sum_{i=1}^{n}(W+t_i)s_i(1-\theta_i)/\theta_i = -W_L L \tag{49}$$

Let $\overline{U}^i = 0, i = 1, 2, \ldots, n$. Then (20) yields:

$$\delta_i\theta_i/[(W+t_i)L_i] = \delta_j\theta_j/[(W+t_j)L_j] \tag{50}$$

Therefore, the share of the total wage bill that will go to union i, $\omega_i = W_iL_i/(WL)$, will be

$$\omega_i = [(W+t_i)L_i]/(WL) = \delta_i\theta_i/\sum_{j=1}^{n}\delta_j\theta_j = (\delta_i/\sum_{j=1}^{n}\delta_j)\theta_i/\overline{\theta} \tag{51}$$

The distribution of labor income among unions given by (49) does not reproduce what, in the previous setting, corresponded to the optimal distribution of employment given by (39). Moreover, union i's share of the total wage bill varies positively with its preference for wage while with uniform wages, it varied positively with its preference for employment.

Union i will have now a higher share of the wage bill than with uniform wages iff its preference for wage is higher than the average, i.e., iff:

$$\theta_i > \overline{\theta} \tag{52}$$

Replacing (49) in (47) and rearranging, we get:

$$\sum_{i=1}^{n}\delta_i(1-\theta_i)/\sum_{i=1}^{n}\delta_i\theta_i = -W_L L/W = 1/\eta \tag{53}$$

or

$$\sum_{i=1}^{n}\delta_i\theta_i/\sum_{i=1}^{n}\delta_i(1-\theta_i) = \overline{\theta}/(1-\overline{\theta}) = \eta \tag{54}$$

(52) reproduces (41). Interestingly, aggregate employment and wage will be in the same point of the labor demand schedule as efficient bargaining. This is due to the particular form of the unions' utility functions.

Using (18),

$$(W + t_i)(1 - \theta_i)/\theta_i - t_i = -W_L L \tag{55}$$

and

$$t_i/W = [\theta_i/\eta - (1 - \theta_i)]/(1 - 2\theta_i) \tag{56}$$

Then, the percentage deviation of unions' i wage from the average wage in the economy is:

$$W_i/W = (W + t_i)/W = \theta_i[1/\eta - 1]/(1 - 2\theta_i) \tag{57}$$

Notice that if $\overline{\theta} < 0.5$, then, from (52), $\eta < 1$; for $W_i/W > 0$, it must be the case that $\theta_i < 0.5$ for all i. If $\overline{\theta} > 0.5$, the reverse must happen. In any case, as long as $W_i/W > 0$, the ratio of a particular union members wage to the average wage increases with the union's preferences for wage.

Using (55) and also (51):

$$\begin{aligned} W_i/W &= (W + t_i)/W = [\theta_i/(1 - 2\theta_i)] \sum_{j=1}^{n} \delta_j(1 - 2\theta_j)/\sum_{j=1}^{n} \delta_j\theta_j \quad (58) \\ &= [\theta_i/(1 - 2\theta_i)]/[\overline{\theta}/(1 - 2\overline{\theta})] \end{aligned}$$

With (49):

$$\begin{aligned} s_i &= L_i/L = \delta_i(1 - 2\theta_i)/\sum_{j=1}^{n} \delta_j(1 - 2\theta_j) \quad (59) \\ &= (\delta_i/\sum_{j=1}^{n} \delta_j)(1 - 2\theta_i)/(1 - 2\overline{\theta}) \end{aligned}$$

The employment share varies negatively with θ_i iff $\theta_i < 0.5$ if all unions show stronger preference for employment than for wage; if not, the reverse happens. One can show that the employment share of union i will be lower than in efficient bargaining with no transfers if (50) holds iff $\theta_i < 0.5$. When $\theta_i > 0.5$ the reverse happens.

Summarizing:

PROPOSITION 3. *In a coalition of n unions with Stone-Geary preferences with the possibility of transfers across employed workers:*

3.1.For positive wages, either $\theta_i > 0.5$ for all i; or $\theta_i < 0.5$ for all i.

3.2.unions with higher relative preference for wage will be allocated a higher share of the total wage bill. The reverse happened when transfers were not allowed [12]*.*

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[12] See Martins and Coimbra (1997b).

3.3.unions with higher relative preference for wage will be allocated a larger share of total wage bill than they were when transfers were not allowed iff they show stronger relative preferences for wage (over employment) than average, i.e., $\theta_i > \overline{\theta}$*. If* $\theta_i < \overline{\theta}$*, the reverse happens.*

3.4.unions with higher relative preference for wage will be allocated a larger share of total employment than they were when transfers were not allowed iff $\theta_i > 0.5$*. If* $\theta_i < 0.5$*, the reverse happens.*

With a linear demand schedule (37), we can deduct from (53) that aggregate employment and wage will be the same as in efficient bargaining, i.e.:

$$L = (a/b)\sum_{i=1}^{n}[(1-\theta_i)\delta_i]/\sum_{i=1}^{n}\delta_i = (a/b)(1-\overline{\theta}) \tag{60}$$

and

$$W = a\sum_{i=1}^{n}\theta_i\delta_i/\sum_{i=1}^{n}\delta_i = a\overline{\theta} \tag{61}$$

Using (58) and (57):

$$L_i = [\delta_i(1-2\theta_i)/\sum_{j=1}^{n}\delta_j(1-2\theta_j),], \tag{62}$$

$$(a/b)\sum_{j=1}^{n}[(1-\theta_j)\delta_j]/\sum_{j=1}^{n}\delta_j = [(\delta_i/\sum_{j=1}^{n}\delta_j)(1-2\theta_i)/(1-2\overline{theta})](a/b)(1-\overline{\theta}), i = 1, 2, \dots, n$$

With (59), (52) and (53):

$$\begin{aligned} t_i &= (\theta_i - \sum_{j=1}^{n}\delta_j\theta_j/\sum_{j=1}^{n}\delta_j)a/(1-2\theta_i) \\ &= (\theta_i - \overline{\theta})a/(1-2\theta_i), i = 1, 2, \dots, n \end{aligned} \tag{63}$$

Proposition 4. *In the case of efficient bargaining among unions, with Stone-Geary union preferences and a linear demand, the equilibrium aggregate employment and (average) wage is invariant to the possibility of wage transfers.*

4.4. Fully Efficient Bargaining with Transfers

Consider that we reproduce condition (29) for the case where $\overline{U}^i = 0, i = 1, 2, \ldots, n$. Then, we get:

$$W_i L_i = [\delta_i \theta_i / (\delta_j \theta_j)] W_j L_j \tag{64}$$

Immediately, we can derive the wage bill share of union i:

$$\omega_i = W_i L_i / \sum_{j=1}^{n} W_j L_j = \delta_i \theta_i / \sum_{j=1}^{n} \delta_j \theta_j = (\delta_i / \sum_{j=1}^{n} \delta_j) \theta_i / \overline{\theta} \tag{65}$$

Union i has the same share as with only efficient bargaining among unions when multiple wage payments are possible (49).

Using (30), we can get:

$$W_i = [\theta_i / (2\theta_i - 1)](a - bL) \tag{66}$$

For positive W and positive marginal product of labor in equilibrium, it must be the case that for all i, $\theta_i > 0.5$.

Then,

$$W_i / W_j = [\theta_i / (2\theta_i - 1)] / [\theta_j / (2\theta_j - 1)] = [\theta_i / (1 - 2\theta_i)] / [\theta_j / (1 - 2\theta_j)] \tag{67}$$

From (63) and (65), we can conclude that:

$$L_i / L_j = [\delta_i (1 - 2\theta_i)] / [\delta_j (1 - 2\theta_j)] \tag{68}$$

Therefore:

$$s_i = L_i / L = \delta_i (1 - 2\theta_i) / \sum_{j=1}^{n} [\delta_j (1 - 2\theta_j)] = (\delta_i / \sum_{j=1}^{n} \delta_j)(1 - 2\theta_i)/(1 - 2\overline{\theta}) \tag{69}$$

This is equal to (57).

Consider we multiply (64) by L_i and sum over i; we get:

$$\sum_{i=1}^{n} W_i L_i = (a - bL) \sum_{i=1}^{n} [\theta_i / (2\theta_i - 1)] L_i \tag{70}$$

Dividing by L to get the average wage in the economy, (31):

$$\sum_{i=1}^{n} W_i L_i / L = W = (a - bL) \sum_{i=1}^{n} [\theta_i / (2\theta_i - 1)] s_i \tag{71}$$

Replacing (67):

$$\sum_{i=1}^{n} W_i L_i / L = W = (a - bL) \sum_{i=1}^{n} \delta_i \theta_i / \sum_{i=1}^{n} [\delta_i (2\theta_i - 1)] \quad (72)$$
$$= (a - bL)[\overline{\theta}/(2\overline{\theta} - 1)]$$

This expression is equal to (45).

The percentage deviation of union's i wage relative to average wage is, therefore using also (64):

$$W_i/W = [\theta_i/(2\theta_i - 1)]/[\overline{\theta}/(2\overline{\theta} - 1)] \quad (73)$$
$$= [\theta_i/(1 - 2\theta_i)]/[\overline{\theta}/(1 - 2\overline{\theta})]$$

This is equal to (56).

PROPOSITION 5. *In the presence of multiple wage payments, if unions exhibit Stone-Geary preferences and labor demand is linear:*

5.1. The relative union positions are preserved when efficient cooperation among unions is enlarged to the employer side.

5.2. The aggregate contract curve over total employment and the average wage in the economy is the same as with uniform wages.

5. SUMMARY AND CONCLUSIONS

This paper contrasts efficient bargaining outcomes in the labor market in scenarios of uniform and multiple wage payments when there is cooperation among the unions in the system, and in the presence of full cooperation between employers and unions.

The main equilibrium conditions for the general (union utility and labor demand) cases are presented in Table 1. It was found that, in general, efficient allocations will be different in the four cases considered. However, distribution conditions of employment (or wage bill) among unions may be similar in uniform wage equilibrium for the two cooperative systems considered. And also in multiple wage systems. Those conditions, however, differ between the two payment systems. The same happens with (general) aggregate efficient conditions.

Using a Stone-Geary utility function and a linear demand schedule, we confirmed the previous inductions and derived additional conclusions see a summary of the results in Table 2:

TABLE 1.

Marginal Rate of Substitution and Distribution Conditions

	Equation	Efficiency Locus
A. Efficient Union Cooperation	(6)	$\sum_{i=1}^{n} U_W^i / U_L^i = -L_W$
B. Fully Efficient Cooperation	(9)	$\frac{\sum_{i=1}^{n} U_W^i}{U_L^i} = \frac{\Pi_W}{\Pi_L} = \frac{-(\sum_{i=1}^{n} L_i)}{[PF_L - W]}$
C. Efficient Bargaining with Transfers	(22)	$\frac{\sum_{i=1}^{n} s_i^2 U_{L_i}^i}{U_W^i} = -W_L$
D. Fully Efficient Bargaining with Transfers	(30)	$\frac{U_W^i}{U_L^i} = \frac{\Pi_{W_i}}{\Pi_{L_i}} = -\frac{L_i}{[PF_L - W_i]}$
	Equation	Distribution Locus
A. Efficient Union Cooperation	(5)	$\frac{\delta_i U_L^i}{[U^i(L_i, W) - \overline{U}^i]} = \frac{\delta_j U_L^j}{[U^j(L_j, W) - \overline{U}^j]}$
B. Fully Efficient Cooperation	(5)	$\frac{\delta_i U_L^i}{[U^i(L_i, W) - \overline{U}^i]} = \frac{\delta_j U_L^j}{[U^j(L_j, W) - \overline{U}^j]}$
C. Efficient Bargaining with Transfers	(16)	$\frac{\delta_i (U_L^i - U_W^i t_i / L_i)}{[U^i(L_i, W+t_i) - \overline{U}^i]} = \frac{\delta_j (U_L^j - U_W^j t_j / L_j)}{[U^j(L_j, W+t_j) - \overline{U}^j]}$
D. Fully Efficient Bargaining with Transfers	(29)	$\frac{\delta_i (U_W^i / L_i)}{[U^i(L_i, W) - \overline{U}^i]} = \frac{\delta_j (U_W^j / L_j)}{[U^j(L_j, W) - \overline{U}^j]}$

TABLE 2.

Equilibrium Solution: Relative Union Positions

	ω_i	s_i	W_i/W
A. Efficient Union Cooperation	$\frac{(\delta_i / \sum_{j=1}^{n} \delta_j)(1-\theta_i)}{(1-\overline{\theta})}$	$\frac{(\delta_i / \sum_{j=1}^{n} \delta_j)(1-\theta_i)}{(1-\overline{\theta})}$	1
B. Fully Efficient Cooperation	$\frac{(\delta_i / \sum_{j=1}^{n} \delta_j)(1-\theta_i)}{(1-\overline{\theta})}$	$\frac{(\delta_i / \sum_{j=1}^{n} \delta_j)(1-\theta_i)}{(1-\overline{\theta})}$	1
C. Efficient Bargaining with Transfers	$\frac{(\delta_i / \sum_{j=1}^{n} \delta_j)\theta_i)}{\overline{\theta}}$	$\frac{(\delta_i / \sum_{j=1}^{n} \delta_j)(1-2\theta_i)}{(1-2\overline{\theta})}$	$\frac{[\theta_i/(1-2\theta_i)]}{[\overline{\theta}/(1-2\overline{\theta})]}$
D. Fully Efficient Bargaining with Transfers	$\frac{(\delta_i / \sum_{j=1}^{n} \delta_j)\theta_i)}{\overline{\theta}}$	$\frac{(\delta_i / \sum_{j=1}^{n} \delta_j)(1-2\theta_i)}{1-2\overline{\theta})}$	$\frac{[\theta_i/(1-2\theta_i)]}{[\overline{\theta}/(1-2\overline{\theta})]}$

1. Wage bill and employment shares change when we allow for multiple wage payments. For example, in multiple wage equilibria, unions' wage bill shares vary positively with own preferences for wage (relative to employment), while with uniform wages the reverse happened. Also, the possibility of wage transfers reinforces the wage bill share of unions that exhibit stronger preferences for wage than average.

2. However, distribution and/or union relative positions are the same in a particular payment system whether partial (i.e., only among unions) or full cooperation is considered.

3. With partial cooperation, possibility of transfers leads to the same aggregate labor market outcome - i.e., wage and total employment - than in its absence.

4. The aggregate contract curve when transfers or differentiated wages are allowed is the same as with uniform wages.

Finally, the arguments advanced in the text may be applicable to analyse a product or any other factor (intermediate product) market equilibrium and market shares arrangements or agreements under firms collusion.

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