

## The Impact of Internet Sales Tax in a Search Model of Money: Some Analytical Results

Tiantian Dai

*China Economics Management Academy, Central University of Finance and  
Economics, Beijing, 100081, China*  
E-mail: dait@econ.queensu.ca

Shenyi Jiang

*China Economics Management Academy, Central University of Finance and  
Economics, Beijing, 100081, China*  
E-mail: Syijiang@gmail.com

Xiangbo Liu\*

*Renmin University of China and International Monetary Institute (IMI), Beijing,  
100872, China*  
E-mail: xiangbo.liu@ruc.edu.cn

and

Wen Wang

*Department of Economics, Duke University, Durham NC 27708, USA*  
E-mail: wen.wang1@duke.edu

We use a search-theoretic model to study the impact of internet sales taxes, in both lump-sum and proportional fashions. We show that both forms of taxes, especially the lump-sum tax, have real effects on the online market if the terms of trade are negotiable, while a proportional tax distorts the economy further. We then propose a preferential tax policy and show that it together with a lump-sum internet sales tax can recover the first best. We also give some policy suggestions.

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## 1. INTRODUCTION

Global electronic commerce sales will increase by 18.3% to \$1.298 trillion in 2013, according to the estimation by eMarketer. Although online transactions currently still make up only a very small fraction of total retail sales, the rapid growth of e-commerce and its de facto tax-free status have kindled a considerable debate surrounding the issue of internet taxation. Some researchers, making an “infant industry” argument, favor no tax or at least no tax in the short run to protect the development of e-commerce.<sup>1</sup> Other researchers argue that if electronic commerce were tax-free, sales tax base would be eroded and traditional retailers would become less competitive in the market.<sup>2</sup> If e-commerce were to be taxed, what is the impact on internet purchases and an individual’s welfare, would taxation seek to be neutral and equitable between e-commerce and traditional commerce or preferential for e-commerce? Surprisingly, no one has provided theoretical answers to these important questions. To fill the gap in the literature, we use a search-theoretic model à la Lagos and Wright (2005) where agents trade both anonymously online (decentralized market) and in the traditional market (centralized market), to study the impact of internet sales tax, in both lump-sum and proportional fashions, and its policy implications. We show that both forms of taxes, especially the lump-sum tax, have real effects if the terms of trade are negotiable. The proportional tax distorts the economy further and can only achieve a third best. Hence, in order to recover the first best in this economy, we show that the preferential tax policy should be conducted, more precisely, subsidizing online buyers.

The online market is partially featured by anonymity and random matching. Its price mechanism includes bargain, auction and price posting. In this paper, we focus on price bargaining, since more people learned how to bargain online and this phenomenon is negligible. For example, China’s biggest e-commerce player, Alibaba, provides tools for buyers and sellers to negotiate prices before trading. Even in US, there are many software online for buyers to use. Therefore, even sellers post prices online, the prices are still negotiable to some extent. We assume the trade is conducted through an intermediary, in reality, they are PayPal in U.S. or Alipay in China for example.<sup>3</sup> Usually, such intermediary is an online payment system which

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<sup>1</sup>The most substantial academic work on the sensitivity of online sales to taxation is Goolsebee (2000). Using the 1997 Forrester Research survey data, he found that consumers living in the high sales tax rate states tend to purchase online and subjecting e-retailers to taxation will reduce online sales by 24%. Also see, Trandel (1992) for similar arguments for the use tax on cross-border sales.

<sup>2</sup>Bruce and Fox (2000) estimate the sales tax revenue losses quantitatively based on the general sales tax and predict that e-commerce may cause about 10.8 billion tax revenue losses nationwide in 2003.

<sup>3</sup>To avoid cheating associated with online transaction, Alibaba start a third warranty, Alipay, to settle payments for buyers. The intermediary grows into an indispensable

holds the payment for the buyers to avoid receiving flawed products sold by suppliers. Since every transaction record can be kept, the government actually can collect the internet sales tax through the intermediary.<sup>4</sup> Using the LW framework, we can show that the existence of such intermediary facilitates trades and improves the individual's welfare (see e.g. He, Huang and Wright, 2005, 2008). Therefore, our analysis is under the assumption that taxing through an intermediary is feasible.<sup>5</sup>

We are also interested in the optimal fiscal policy that can protect the development of e-commerce in the infant period as well as improve the social welfare. We show that granting a tax exemption is not enough to recover the first best and government can conduct a proportional subsidy policy for online buyers. The optimal amount of subsidy is not trivial, which depends on the quantity of money traded. In particular, the more money a buyer carries, the more she should be subsidized. This is because, first, the subsidy can compensate the loss in the total surplus caused by the internet sales tax. Second, since money is also subject to the inflation tax, large money holders should receive more subsidies in order to cover these losses. In general, the source of the subsidy is not limited in general, while it collected from internet sales tax in our model.

In terms of related works, Bruce, Fox and Murray (2000) argue that the optimal tax literature does not support exemption of internet sales tax in general since the optimal conditions are hard to be met in practice. McLure, Jr. (2003) proposed an economically efficient sales tax system for e-commerce in which all sales to customer would be taxed, all sales to business would be exempted while sales by local and remote vendors would be taxed equally. Ellison and Ellison (2009) estimate the sales of a group of small firms selling computers parts online and find that e-retail sales are very sensitive to taxes levied on traditional retail purchases. However, supporters of preferential tax treatment on e-commerce provide some compelling arguments that the e-commerce market tends to under-provide goods characterized by network externalities if treated equally (Zodrow, 2003). Similarly, Goolsbee and Zittrain (1999) note that the existence of various network externalities may imply a text "significant social cost" if

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tool for online transaction in China. In 2012, the total transactions through Alipay on a single bachelor's day exceeded 3 billion US dollars.

<sup>4</sup>The anonymity of online transactions seriously complicate both tax administration and tax compliance, if taxes are based on the destination of sales or the source of income. With proper administration and technology, the existence of internet tax intermediary can solve this problem.

<sup>5</sup>In the literature, an intermediary is identified with four roles: aggregation, pricing, search, and trust. The intermediary in our model aggregates all the information among suppliers, offers the platform for searching and transaction and holds money for buyers. We do not explore the role of pricing. The roles of aggregating and trust are also different from those in the previous literature (see among others, Williamson, 1975, Croson, 1995 and Buxmann, Rose, and Konig, 1997).

computer network was inefficiently small. In this case, preferential tax treatment of e-commerce might be desirable to encourage the expansion of the network to its efficient size, at least in the short run. Network externalities thus potentially supply an defensible economic rationale for preferential tax treatment of e-commerce. Nevertheless, all these arguments are based on the changes of tax structure to compensate the revenue loss as well as to protect the infant industry. Contrary to their works, we argue that a preferential tax policy can offset the negative effect of internet sales tax.

The rest of the paper is organized as follows. Section 2 describes the baseline model with taxation. Section 3 discusses the preferential tax policy. Section 4 gives out some political suggestions. Section 5 concludes.

## 2. BASELINE MODEL

### 2.1. Environment

Time is discrete. A  $[0, 1]$  continuum of agents live forever with discount factor  $\beta \in (0, 1)$ . Each period is divided into two subperiods. In the first subperiod, a decentralized online market opens, agents trade special goods  $q$  anonymous online. Agents are matched randomly online with  $\sigma$  being the probability of single coincidence of wants, where  $\sigma \in (0, \frac{1}{2})$ . With probability  $1 - 2\sigma$ , agents are non-traders. In each match, buyers enjoy utility  $u(q)$  while sellers suffer disutility  $c(q)$ . Functions  $u$  and  $c$  are twice continuously differentiable  $u', c' > 0, u'' < 0, c'' \geq 0, u(0) = c(0) = 0$ . In order to rule out barter trades, we assume that there is no double coincidence of wants. We assume that the payment is settled through a benevolent intermediary. In particular, the intermediary holds buyers' payments before they receive the right products.<sup>6</sup> Therefore, this intermediary can collect internet sales taxes for the government who can also pay subsidies back to the traders through this system. Nevertheless, we do not explicitly model the intermediary here. In the baseline model, we assume that government levies both a lump-sum tax ( $T$ ) and a proportional internet sales tax with  $t_d$  being the tax rate.

In the second subperiod, a centralized market opens where agents engage in traditional trades. They produce and consume a general good. The production technology is a one for one transformation from labor  $H$  into a general good. The utility function is quasi-linear,  $U(X) - H$  with  $U' > 0 \geq U''$ . Both special goods and the general good are not storable. Therefore, fiat money is the only object which can be used as a medium of change in this model.

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<sup>6</sup>In reality, in order to avoid buyers from cheating, the intermediary sets a deadline to buyers for confirming their orders. Therefore, sellers can get the payment as long as buyers do not choose to return their orders.

The aggregate money supply evolves according to  $M_{+1} = (1 + \tau)M$ , where  $+1$  denotes the next period. Let  $\phi$  be the price of money in terms of goods. Then the government budget constraint is  $G = T\phi + t_d d\phi + \tau M\phi$ , where  $G$  is the government spending. Note here, we assume that government charges different  $T$  every period such that keeping the real lump sum tax ( $T\phi$ ) being constant over time. The newly printed money are injected through a lump sum transfer to each agent during the second period.

## 2.2. Agent's problem

Let  $W(m)$  be the value function for an agent in the centralized market and  $V(m)$  be the value function in the decentralized market, where  $m$  is an agent's money holding. Then an agent's problem in the centralized market is

$$W(m) = \max_{X, H, m_{+1}} \{U(X) - H + \beta V_{+1}(m_{+1})\}, \quad (1)$$

subject to

$$X = H + \phi(m - m_{+1}) + \tau M\phi. \quad (2)$$

As standard, we can get two first order conditions  $U'(X^*) = 1$  and  $\beta V'_{+1, m}(m_{+1}) = \phi$ , which implies  $X^* = X^{FB}$  and the money distribution is degenerate at the end of each period.

Moving to the decentralized market, agents are random matched with the payoff

$$\begin{aligned} V(m) = & \sigma[W(m - (1 + t_d)d - T) + u(q)] \\ & + \sigma[W(m + d) - c(q)] + (1 - 2\sigma)W(m). \end{aligned} \quad (3)$$

The first term is the flow value of a matched buyer who pays  $(1 + t_d)d - T$  quantity of money for  $q$  quantity of goods.<sup>7</sup> The second term is the flow value of a matched seller who receives money and pays disutility of producing. Lastly, the third term is the value of being a non-trader.

The terms of trade  $(q, d)$  is determined by Nash bargaining with  $\theta$  being the buyer's bargaining power. We denote the buyers' and sellers' money holding as  $m_b$  and  $m_s$  respectively. Hence, the buyer's trading surplus is  $u(q) - ((1 + t_d)d + T)\phi$ , and the seller's trading surplus is  $-c(q) + d\phi$ . As in Lagos and Wright (2005), we can show that, in equilibrium, agents would not bring unused money into the decentralized market ( $d = m$ ), therefore

<sup>7</sup>Since the terms of trade are determined by the bargaining process, our results will not change qualitatively if taxes are passed forward to sellers.

we can get  $m\phi = g(q)$  and  $g'(q)q'(m) = \phi$ , where,

$$g(q) = \frac{\theta c(q)u'(q) + (1 - \theta)c'(q)[u(q) - \phi T]}{\theta u'(q) + c'(q)(1 - \theta)(1 + t_d)}, \quad (4)$$

and

$$g_q = \frac{u'c'[\theta u' + (1 - \theta)(1 + t_d)c'] + \theta(1 - \theta)[u - (1 + t_d)c - \phi T](u'c'' - c'u'')}{[\theta u' + (1 - \theta)(1 + t_d)c']^2} > 0. \quad (5)$$

Since the total trading surplus  $u(q) - c(q) - t_d d\phi + T\phi$  and the seller's surplus  $-c(q) + d\phi$  are all non-negative, we can show that  $u - (1 + t_d)c - \phi T \geq 0$ , and, hence  $g_q > 0$  follows. Then by using bargaining solution and repeated substitution, we can get

$$g(q) = \beta g(q_{+1}) \left\{ \sigma \left[ \frac{u'(q_{+1})}{g'(q_{+1})} - (1 + t_d) \right] + 1 \right\}. \quad (6)$$

Then, we assume there is a unique equilibrium in this model,<sup>8</sup> and the steady state equation is

$$\frac{u'(q)}{g'(q)} = \frac{1 - \beta}{\beta\sigma} + (1 + t_d). \quad (7)$$

### 2.3. The Impact of Internet Taxes

In this section, we study the impact of two types of internet taxes, namely, lump-sum tax and propotional tax. By taking the total derivatives, we can show that both types of taxes have negative effects on the quantity of goods traded in the online market. Intuitively, a higher propotional tax rate reduces buyers' real money balances, which lowers their total trading surplus. This discourages buyers' incentives from bringing money into the online market. As a result, sellers produces less, and the quantity of goods per match decreases. The more interesting result is the novel effect of lump sum tax, since it does not affect agents' decisions in a traditional model. The lump sum tax has a negative effect in this model is because of  $T\phi$  entering the bargaining problem and  $m = d$  in equilibrium. Since buyers are constrained, a higher lump sum tax reduces buyers' real money balances, and hence buyers would bring less money into the market, which has a negative effect on the quantity of goods per match.<sup>9</sup> We summarize these results in the following proposition.

<sup>8</sup>Actually,  $g''(q) > g'(q)u''/u'$  is a sufficient condition for the uniqueness.

<sup>9</sup>If buyers bring enough money in the the online market, we will get  $u'(q) = c'(q)$ , and  $d = [\theta c(q) + (1 - \theta)u(q)] / [\phi(1 + t_d)] - (1 - \theta)T / (1 + t_d)$ . Therefore,  $\phi T$  do not have real effects on  $q$ .

PROPOSITION 1. *The quantity of good per match traded in the online market decreases in both lump sum tax and propotional tax rate.*

*Proof.* Differentiating the steady state equation with respect to  $t_d$  and  $T\phi$ , we can get

$$\frac{dq}{dt_d} = \frac{(g'(q))^2}{u''(q)g'(q) - 2g''(q)u'(q)} < 0. \quad (8)$$

$$\frac{dq}{d(T\phi)} = \frac{g''(q)u'(q)}{u''(q)g'(q) - g''(q)u'(q)} < 0. \quad (9)$$

Note that  $u''(q) < 0$ ,  $u'(q) > 0$  and  $g'(q) > 0$ . Moreover,  $g''(q) > 0$ , because in equilibrium the money constraint is always binding (ex.  $m = d$ ). Thus, the quantity of goods per match cannot achieve the efficient level, and  $g(q)$  should increase with an increasing rate. ■

As showed in Lagos and Wright (2005), the first-best outcome is in general not attainable and depends on the bargaining power, discount factor and the money growth rate. Here, from the bargaining solution, we have  $u'(q) = c'(q)(1 + t_d)$ , which shows that a proportional tax distorts agents' decisions and reduces the online market's efficiency further. Now, we know that if there exists bargaining in the online market, the development of this market can be limited by internet sales taxes, while tax exemption does hurt the traditional retailers on the other hand. Therefore, we are in a position to discuss whether a preferential tax policy can balance the two. Since a positive tax rate distorts the online market further, we will only consider the preferential tax policy with a lump-sum internet sales tax.

### 3. PREFERENTIAL TAX POLICY

The rationale for preferential taxation in the decentralized markets can be explained with network externalities<sup>10</sup>, environmental externalities<sup>11</sup>

<sup>10</sup>A network externality (sometimes called a "network effect") is the cost or benefit that incumbent users get from an additional member joining the network. Katz and Shapiro (1985; 1994) divided these benefits into two types. It can be direct (such as the benefit from having one more person to exchange information on the product) or indirect (from a larger network of users encouraging greater investment in network resources and increase the choices for customers).

<sup>11</sup>A physical presence requires selection of a proper location with the facilities necessary to serve consumers while an Internet server can be in a very remote location and with no physical storefront, which means less space occupation and less waste.

and efficiency improvement<sup>12</sup>. Though we do not explicitly model these externalities, they do affect agents' decisions in the real world. With an efficient market size, the cost of peripheral services can be much lower.<sup>13</sup> Therefore, we argue that preferential tax can potentially reduce the effects of those negative externalities.

The setup for the centralized market is the same as in the baseline model. The difference is that, in the online market, if a buyer matched with a seller, we assume that he will receive a subsidy  $B(d)$  which depends on the quantity of money traded. Therefore, the new value function is

$$\begin{aligned} V(m) = & \sigma[W(m-d-T+B(d))+u(q)] \\ & + \sigma[W(m+d)-c(q)] + (1-2\sigma)W(m). \end{aligned} \quad (10)$$

We will show that  $B(d)$  is not trivial, namely, it is not a simple lump-sum transfer. The terms of trade  $(q, d)$  is determined by solving the following Nash bargaining problem

$$[u(q) + W(m_b - d - T + B(d)) - W(m_b)]^\theta [-c(q) + W(m_s + d) - W(m_s)]^{1-\theta}. \quad (11)$$

Still, we have  $d = m$  in equilibrium, and it is easy to check that  $m < m^*$ , with  $m^*$  being the efficient money holding (ex.  $u'(q) = c'(q)$ , if  $m \geq m^*$ ). Therefore we can get  $m\phi = g(q)$  and  $g'(q)q'(m) = \phi$ , where,

$$g(q) = \frac{\theta c(q)u'(q) + (1-\theta)c'(q)[u(q) - \phi T + \phi B(m)]}{\theta u'(q) + c'(q)(1-\theta)}, \quad (12)$$

and

$$g_q = \frac{u'c'[\theta u' + (1-\theta)c'] + \theta(1-\theta)[u - c - \phi T + \phi B(m)](u'c'' - c'u'')}{[\theta u' + (1-\theta)c']^2} > 0. \quad (13)$$

It is easy to check that  $u - c - \phi T + \phi B(m) \geq 0$ . Then by using bargaining solution and repeated substitution, we can show that the slope of  $V(m)$  as  $m \rightarrow m^*$  is proportional to the equation below

$$-\phi + \beta\phi_{t+1}\{\sigma\Gamma + 1\}, \quad (14)$$

<sup>12</sup>When markets transition from a physical environment to the Internet, consumer who has a limited set of choices because of geographical limitation and search costs can have more choices and convenience. The greater competition in the market, the greater choice of suppliers and product selection for consumers make trade easier and efficient. Another benefit in electronic commerce is the electronic player is infinitely patient and customer-led.

<sup>13</sup>For e-commerce, it includes fixed cost, shipping costs and storage costs, which all contribute to the benefits of economies of scale.

where

$$\Gamma = u' \frac{1}{g'_{+1}} - 1 + B'(m). \quad (15)$$

$\Gamma$  is the buyer's marginal benefit of bringing an additional dollar evaluated at  $q = q^*$  and also equals

$$\Gamma = \frac{c'^2}{c'^2 + \theta(1 - \theta)[u - c - \phi T + \phi B(m)][c'' - u'']} + B'(m) - 1. \quad (16)$$

As showed in Lagos and Wright (2005), only in the extreme case where  $\phi_t = \beta\phi_{t+1}$  and  $\theta = 1$ , the first best can be achieved (ex.  $m = m^*$ ). Contrary to their results, we will show that the first best is achievable with  $B(m)$ , with the optimal inflation, even if  $\theta = 1$ . First, we can show that  $B'(m) > 0$ ,  $B''(m) < 0$ , and  $B'(m) \rightarrow 0$  as  $m \rightarrow m^*$ . These conditions imply that the more buyers spent in the online market, the more subsidies he can get; and the diminishing increase of subsidy implies that the economy is achieving the first best.<sup>14</sup> Therefore, if  $\theta = 1$ , we can show that  $B'(m) = 0$  at the Friedman rule (ex.  $\Gamma = 0$ ), which implies  $m = m^*$ . This condition states that if the buyer has all the bargaining power, the government, in order to recover the first best, has to subsidize the buyer  $B(m^*)$  amounts of money such that  $B'(m^*) = 0$ . Next, if  $\theta < 1$ ,  $\Gamma = 0$  implies  $B'(m) > 0$ , therefore, the first best is still achievable, if  $B(m)$  satisfies the following condition

$$B'(m)c'^2 + \theta(1 - \theta)[u - c - \phi T + \phi B(m)][c'' - u''] [B'(m) - 1] = 0. \quad (17)$$

Note that the buyers receives less subsidy in this case. The intuition is the following. The bargaining solution only depends on the buyer's money holding, and he brings less money into the online market if the seller shares the trade surplus, and, thus, the government do not need to rebate as much as in the  $\theta = 1$  case. The above analysis can be summarized in the following Lemma and Proposition.

LEMMA 1. *The subsidy function is locally concave, in particular,  $B'(m) > 0$  and  $B''(m) \leq 0$  as  $m \rightarrow m^{**}$  (see Appendix for a proof).*

PROPOSITION 2. *The the first best can be recovered with a preferential tax on buyers at the Friedman rule. In particular, the first best requires  $B'(m) = 0$  if  $\theta = 1$  and requires  $B'(m^{**})c'^2 + \theta(1 - \theta)[u - c - \phi T + \phi B(m)][c'' - u''] [B'(m^{**}) - 1] = 0$  if  $\theta < 1$ , where  $m = d$  in equilibrium.*

<sup>14</sup>We also checked other government policies, such as the subsidies on the sellers, lump-sum subsidies on the buyers and subsidies both buyers and sellers, but none of them can recover the first best.

#### 4. POLICY IMPLICATIONS

Here we discuss some policy implications that implied by our results. First, we show that a lump sum internet sales tax can reduce the quantity of goods traded in the online market, if buyers can bargain the price. This implies that the lump sum tax may have real effects, and the results depends on the market structure. Of course, people do observe other price mechanisms existing in the online market, such as price posting and auction. We believe that comparing the effects of taxes under different price mechanism, both qualitatively and quantitatively, are very interesting and important, but this is beyond the scope of this paper.

Second, we show that a lump sum tax together with a preferential tax policy can recover the first best. Therefore, we suggest that the government can tax the online market as well as protect the market as a infant industry. Moreover, the subsidy is not trivial in general and depends on the quantity of money traded which is equal to the buyer's money holding in this model. Third, non-trivial subsidy depends on the buyer's bargaining power. The more bargaining power a buyer has, the more government has to subsidize. Finally, agents are heterogeneous in the real life, and, therefore large buyers should be subsidized more. Since agents are identical in our model, they get the same amount of subsidy.

#### 5. CONCLUSION

In this paper, we study the impact of internet sales tax on the online market, and find that the lump-sum tax has real effects on agents' decisions if buyers can bargain prices. The quantity of trade per match decreases with both forms of taxes, and the economy can be distorted further by a proportional tax. Given both forms of taxes have negative effect on agents' welfare, we propose a preferential tax and show that itself together with a lump-sum transfer can recover the first best and balance the electronic and traditional commences. Moreover, the preferential tax is not trivial and depends on the quantity of money traded. Aruoba, Boragan and Christopher (2011) use a search model with taxes to study the effect of money on capital, while they tax the activities in the centralized market and answer different questions.

In general, the source of subsidy is not limited to the internet sales tax collected from buyers. Therefore, interactions among different forms of taxes worth further investigating. What do an optimal internet taxation and optimal tax structure look like? How does the relationship between

fiscal policy and monetary policy change if we take the internet sales tax into account? We leave all these open questions for future research.

### APPENDIX: PROOF OF LEMMA 1

Now the value function in the decentralized market still can be reduced to equation (25), while

$$v = \sigma[u(q) + \phi B(m) - \phi d - \phi T] + \sigma[\phi d - c(q)] + U(X^*) - X^*. \quad (\text{A.1})$$

Moreover, the first order condition and the Envelop condition becomes

$$\phi = \delta[v'_{+1} + \phi_{+1}], \quad (\text{A.2})$$

$$v_m = \sigma[u'(q)q' - \phi d' + \phi B'(m)]. \quad (\text{A.3})$$

Again, the slope of equation (11) as  $m \rightarrow m^{**}$  is proportional to the equation below

$$-\frac{g}{m} + \beta\left\{\sigma\left[u' \frac{g_{+1}}{g'_{+1}m_{+1}} + \frac{g_{+1}}{m_{+1}}(B'(m) - 1)\right] + \frac{g_{+1}}{m_{+1}}\right\}, \quad (\text{A.4})$$

where

$$\Gamma = u' \frac{1}{g'_{+1}} - 1 + B'(m). \quad (\text{A.5})$$

$\Gamma$  is the buyer's marginal benefit of bringing an additional dollar evaluated at  $q = q^{**}$  and also equals

$$\Gamma = \frac{c'^2}{c'^2 + \theta(1 - \theta)[u - c - \phi T + \phi B(m)][c'' - u'']} + B'(m) - 1. \quad (\text{A.6})$$

With  $u' > 0$  and  $g_q > 0$ , it is easy to get that  $B'(m^{**}) < 1$  for  $B'(m^{**}) = 1 - \frac{u'}{g_{+1}'}$ . We have already get that the best subsidy structure can be rewritten as  $B'(m^{**})c'^2 + \theta(1 - \theta)[u - c - \phi T + \phi B(m)][c'' - u''][B'(m^{**}) - 1] = 0$  at  $m = m^{**}$ . It is obvious that  $B'(m) > 0$  as  $m \rightarrow m^{**}$ .

The slope of the objective function as  $m \rightarrow m^{**}$  is proportional to the equation below

$$\Gamma = u' \frac{1}{g'_{+1}} - 1 + B'(m). \quad (\text{A.19})$$

Differentiate  $\Gamma$  with respect to  $m$ , we can get that

$$\Gamma' = \frac{u''g'_{+1} - u'g''_{+1}}{(g'_{+1})^2} + B''(m) \quad (\text{A.20})$$

With the assumption of unique equilibrium,  $\Gamma' \leq 0$  should be satisfied for  $m \in [0, m^{**}]$ , then  $B''(m) \leq 0$  must be satisfied as  $m \rightarrow m^{**}$

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