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Inside Trading when the Market Deviates from the Semi-strong Efficient Condition*

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We study the impacts of shared information and price deviation from the semi-strong efficient condition on traders' trading behavior in the context of Kyle (1985)'s speculative market. We find that when the price is lower than the semi-strong efficient price, the insider and outsiders trade more aggressively using their private information, with a result that more information is incorporated into the price. Moreover, both the insider and outsiders prefer the price to be lower than the semi-strong efficient condition, whereas market makers prefer the price to be higher than the semi-efficient condition.

Key Words: Market makers; Outsiders; Inside trading; Price deviation from semi-strong efficient condition; Nash equilibrium.

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1. INTRODUCTION

Using an extension of the framework of Kyle (1985), this paper analyzes the impacts of price deviation from semi-strong efficient condition on the trading behavior of the insider and informed outsiders. Kyle (1985) investigates the problem of how private information is incorporated into public price in a semi-strong efficient speculative market. Kyle (1985) finds that the monopolistic insider, in order to maximize the conditional profit, will trade in a recursive manner in a discrete model; in addition, in the continuous time case, as the time interval goes to zero, the private information is incorporated into the market price at a constant speed, and the market depth is constant over time (For the case of more than two insiders having the same private information, see Holden and Subrahmanyam [1992] and Gong and Liu [2012], who find that each trader tries to beat the others in the market and that their information is revealed almost immediately).

Kyle (1985) has elicited a large body of literature. Caldentey and Stacchetti (2010) study Kyle's (1985) extended model with an insider observing a signal that tracks the evolution of an asset's fundamental value with a random public announcement time revealing the current value of the asset. Luo (2001) extends Kyle's model (without the outsiders) by showing that when there exists public information, the monopoly insider puts negative weight on the public information in formulating his/her optimal strategy by moving the price in the direction of the asset value. Gong and Zhou (2010) improve Kyle's (1985) model by losing the assumption of the constant pricing rule and provide a new framework to analyze the insider's behavior. Some scholars have studied the models with the presence of outsiders who share some information with the insiders (Zhang, 2008; Liu and Zhang, 2011; Grégoire and Huang, 2012). Jain and Mirman (2000), Daher and Mirman (2007), and Wang *et al.* (2009), among others, explored various types of speculative markets by modeling the financial and real sectors together in order to study the insider's (or insiders') decision-making process and its effects on the output in the real sector, stock price of the firm in the financial sector, and information revealed to the public. In addition, Rochet and Vila (1994), Huddart *et al.* (2001), Decamps and Lovo (2006), and Jiang and Shi (2006) have used variants of Kyle's model to analyze and explain the real financial phenomena.

A common characteristic of the studies mentioned above is that they are all based on the assumption that price satisfies the semi-strong efficient condition. However, several scholars have studied the efficiency of the financial market from different perspectives using different methods and models (Longworth *et al.*, 1981; Rozeff and Zaman, 1988). Although the results are varied, they all agree that some markets do not meet the semi-strong efficient condition (see Givoly and Lakonishok [1979], for example). Therefore, it is interesting to consider inside trading when the market deviates from the semi-strong efficient condition.

In this study, we provide a model with shared information when the market deviates from the semi-strong efficient condition. We study the impact of shared information on the behavior of the insider and informed outsiders in a speculative market with four types of traders: one risk neutral insider, M risk neutral outsiders, noise traders, and competitive risk neutral market makers. We find that when the price is lower than the semi-strong efficient price, the insider and outsiders trade more aggressively on their privat information, with a result that more information is incorporated into the price. Moreover, both an increasing number of informed outsiders and the existence of shared information result in more effectiveness of the equilibrium price. Furthermore, it is interesting that both the insider and outsiders prefer the price to be lower than the semi-strong efficient condition, whereas market makers prefer a higher price than the semi-strong efficient case.

This paper is structured as follows. In Section 2, we present the model. In Section 3, we identify the unique linear Nash equilibrium of the model. Then, in Section 4 we analyze the properties of the equilibrium. Section 5 concludes the paper, and the Appendix provides proofs.

2. THE MODEL

We consider a model with four types of traders: one risk neutral insider (the informed trader), $M(M \geq 2)$ risk neutral outsiders, noise traders, and competitive risk neutral market makers. In addition, there are two periods (period 0 and period 1) and a single risky asset in the economy. At period 0, the public and private information are released and trading takes place, and at period 1, the risky asset payoff is realized. The ex post liquidation value of the risky asset is a random variable $\tilde{v} = \tilde{\zeta} + \tilde{s}$, which is normally distributed with mean p_0 and variance σ_v^2, that is, $\tilde{v} \sim N(p_0, \sigma_v^2)$. The first component, $\tilde{\zeta}$, is related to the private information known only to the insider. Prior to trading, the insider learns the value of security by observing the signal $\tilde{\zeta}$ and $\tilde{s}$. The second component, $\tilde{s}$, is related to the shared information obtainable by every outsider (but not by market makers and noise traders). Every outsider can observe $\tilde{s}$ at time 0, which is drawn from an independent and normal distributed with zero mean and variance $t_s\sigma_v^2$.[1] Thus, the insider's information is $(\tilde{v}, \tilde{s})$, and every outsider's information is $\tilde{s}$. Noise traders have an inelastic demand for the risky asset, and their trading is exogenous. The quantity traded by noise traders,

[1]We assume that $0 < t_s < 1$. The notation of variance used here is for latter computational convenience.

denoted by $\tilde{u}$, is normally distributed with mean zero and variance σ_u^2. The random variables $\tilde{\zeta}$, $\tilde{u}$ and $\tilde{s}$ are mutually independent.

The trading proceeds the same as in Kyle (1985). At period 0, the private information is announced. After receiving the information, the insider chooses his/her trading strategy by submitting an order $\tilde{x} = X(\tilde{v}, \tilde{s})$, and each outsider $i \in \{1, \dots, M\}$ chooses his/her trading strategy by submitting an order $\tilde{y}_i = Y_i(\tilde{s})$. In addition, the trading volume of noise traders, $\tilde{u}$, is realized. The insider and every outsider choose the quantity they trade based on their information. When doing so, they can observe their individual information but do not know $\tilde{u}$ and the others' quantities traded. The market makers observe the total order $\tilde{y} = \tilde{x} + \tilde{y}_1 + \tilde{y}_2 + \cdots + \tilde{y}_M + \tilde{u}$ but not each individual's order. After receiving the total order $\tilde{y}$, the market makers take the opposite side of the incoming order and set the price $\tilde{p} = P(\tilde{y})$ of the risky asset in a bias semi-strong efficient way.[2] At period 1, the uncertainty is resolved and the risky asset payoff is realized. The structure of the economy is common knowledge. We will provide the definition of equilibrium in the following section.

3. THE UNIQUE LINEAR EQUILIBRIUM

In this section, we present the equilibrium concept of the model and state the existence of a unique linear equilibrium.

A strategy for the informed insider is given by a measurable function $X : R^2 \to R$, which determines his/her market order as a function of the insider's observable information. For a given strategy X, the insider's corresponding demand of the asset will be $\tilde{x} = X(\tilde{v}, \tilde{s})$.

For an outsider $i \in \{1, \dots, M\}$, the strategy is given by a measurable function $Y_i : R \to R$, which specifies his/her market order as a function of the outsider's available information. For a given strategy Y_i, let $\tilde{y}_i = Y_i(\tilde{s})$. A strategy combination $(X, Y_1, Y_2, \dots, Y_M)$ determines the order flow as

$$\tilde{y} = \tilde{x} + \tilde{y}_1 + \tilde{y}_2 + \cdots + \tilde{y}_M + \tilde{u}.$$

The market makers observe the realization of the order flow but not of any of its components, and they engage in a competitive auction to serve the order flow. The outcome of this competition is described by a measurable function $P : R \to R$, which specifies the pricing rule that bring them zero expected profit. Given $(P, X, Y_1, Y_2, \dots, Y_n)$, denote $\tilde{p} = P(\tilde{y})$ and let $\tilde{\pi}(X, P) = (\tilde{v} - \tilde{p})\tilde{x}$ and $\tilde{\pi}_i(Y_i, P) = (\tilde{v} - \tilde{p})\tilde{y}_i$ denote the resulting trading profit of the insider and that of the ith outsider, respectively.

[2]It is worth noting that the expected profit of market makers is not zero. We will present the analysis in section 4.3.

DEFINITION 3.1. $(P, X, Y_1, Y_2, \ldots, Y_M)$ is an equilibrium for the one-shot model if

(1) Profit maximization: For any alternate trading strategy X' of the insider,

$$E[\tilde{\pi}(X, P)|\tilde{v}, \tilde{s}] \geq E[\tilde{\pi}(X', P)|\tilde{v}, \tilde{s}].$$

For any alternate trading strategy Y_i' of outsider i,

$$E[\tilde{\pi}_i(Y_i, P)|\tilde{s}] \geq E[\tilde{\pi}_i(Y_i', P)|\tilde{s}],$$

where $i = 1, 2, \ldots, M$.

(2) Bias semi-efficient market efficiency: $P(\tilde{y}) = (1+k)E(\tilde{v}|\tilde{y})$, where k is an exogenous variable.

DEFINITION 3.2. We consider an equilibrium to be a linear equilibrium if the strategy functions $X, Y_i, i = 1, \ldots, M$, and P are all affine functions. That is, there exist constants $a, b_i, c, \alpha, \beta, \gamma_i$, and λ such that

$$\tilde{x} = X(\tilde{v}, \tilde{s}) = a + \alpha\tilde{v} + \beta\tilde{s}, \tag{1}$$

$$\tilde{y}_i = Y_i(\tilde{s}) = b_i + \gamma_i\tilde{s}, \ i = 1, \ldots, M, \tag{2}$$

$$\tilde{p} = P(\tilde{y}) = (1+k)(c + \lambda\tilde{y}). \tag{3}$$

We focus on the linear Nash equilibrium to avoid technical inconvenience due to problems associated with higher-order expectation (forecast the forecast of others and so on), and we find the equilibrium satisfying the following:

THEOREM 1. *When* $k > \frac{-(M+2)^2 + M^2 t_s}{2(M+2)^2 + 2M(M+2)t_s}$,[3] *there exists a unique linear Nash equilibrium given by Eqs. (1)-(3), where*

$$a = -\frac{1+2k}{2\lambda(1+k)}p_0, \tag{4}$$

$$b_i = -\frac{k}{\lambda(1+k)}p_0, \tag{5}$$

[3]The inequality condition ensures that the degree of the deviation is not so big for the equilibrium to exist.

$$c = \frac{1 + (M+2)k}{1+k} p_0, \tag{6}$$

$$\alpha = \frac{1}{2\lambda(1+k)}, \tag{7}$$

$$\beta = -\frac{M}{2\lambda(1+k)(M+2)}, \tag{8}$$

$$\gamma_i = \frac{1}{\lambda(M+2)(1+k)}, \tag{9}$$

$$\lambda = \frac{\sigma_v}{2\sigma_u(1+k)(M+2)} \sqrt{(M+2)^2(1+2k) + (2kM^2 + 4Mk - M^2)t_s}, \tag{10}$$

$$\tilde{y} = \tilde{x} + \tilde{y}_1 + \tilde{y}_2 + \cdots + \tilde{y}_M + \tilde{u}.$$

The proof of this theorem is presented in the appendix.

4. PROPERTIES OF THE LINEAR EQUILIBRIUM

In this section, we analyze the effect of the exogenous variables on trading behavior, pricing rule, profits of the traders, and market depth. The information role of the asset price is also investigated.

4.1. Trading intensity and market depth

Eqs. (7) and (8) imply that $\alpha > 0$, $\beta < 0$, which means that the insider puts a positive weight on private information $\tilde{v}$ but a negative weight on shared information $\tilde{s}$. Moreover, from Eqs. (7)-(9) we also get $\alpha + \beta = \gamma_i$. As $\tilde{v} = \tilde{\zeta} + \tilde{s}$ and the insider observes information $(\tilde{v}, \tilde{s})$, we can equivalently say that the insider knows $(\tilde{\zeta}, \tilde{s})$. The equilibrium trading strategy of the insider can also be expressed as the sum of independent parts, $\tilde{x} = X(\tilde{\zeta}, \tilde{s}) = a + (\alpha + \beta)\tilde{s} + \alpha\tilde{\zeta}$. This means that the insider and every outsider put the same positive weight γ_i on shared information $\tilde{s}$. The insider trades in this way to camouflage the trading on private information.

From the expressions of γ_i and λ in Theorem 1, we have

$$\gamma_i = \frac{2\sigma_u}{\sigma_v \sqrt{(M+2)^2(1+2k) + (2kM^2 - M^2 + 4Mk)t_s}}.$$

The quantity $\frac{1}{\lambda}$ measures the "depth" of the market, that is, the order flow necessary to induce the price to rise or fall by one unit.

From the expressions of α, λ and η in Theorem 1, it is easy to see the following proposition.

PROPOSITION 1. *The comparative statics about variables related to trading intensities and market depth are presented as follows.*

(1)

$$\frac{\partial\alpha}{\partial M}\begin{cases} <0 & if \quad k>\dfrac{M}{M+2}, \\ \geq 0 & if \quad \frac{-(M+2)^2+M^2t_s}{2(M+2)^2+2M(M+2)t_s}<k\leq\frac{M}{M+2}. \end{cases} \tag{11}$$

$$\frac{\partial\alpha}{\partial t_s}\begin{cases} <0 & if \quad k>\dfrac{M}{2(M+2)}, \\ \geq 0 & if \quad \frac{-(M+2)^2+M^2t_s}{2(M+2)^2+2M(M+2)t_s}<k\leq\frac{M}{2(M+2)}. \end{cases} \tag{12}$$

In addition,

$$\frac{\partial\alpha}{\partial k}<0, \qquad \frac{\partial\alpha}{\partial\sigma_u}>0, \qquad \frac{\partial\alpha}{\partial\sigma_v}<0. \tag{13}$$

(2)

$$\frac{\partial\lambda}{\partial M}\begin{cases} >0 & if \quad k>\dfrac{M}{M+2}, \\ \leq 0 & if \quad \frac{-(M+2)^2+M^2t_s}{2(M+2)^2+2M(M+2)t_s}<k\leq\frac{M}{M+2}. \end{cases} \tag{14}$$

$$\frac{\partial\lambda}{\partial t_s}\begin{cases} >0 & if \quad k>\dfrac{M}{2(M+2)}, \\ \leq 0 & if \quad \frac{-(M+2)^2+M^2t_s}{2(M+2)^2+2M(M+2)t_s}<k\leq\frac{M}{2(M+2)}. \end{cases} \tag{15}$$

$$\frac{\partial\lambda}{\partial k}\begin{cases} >0 & if \quad \dfrac{-(M+2)^2+M^2t_s}{2(M+2)^2+2M(M+2)t_s}<k<\dfrac{2Mt_s+2M^2t_s}{(M+2)^2+M(M+2)t_s}, \\ \leq 0 & if \quad k\geq\frac{2Mt_s+2M^2t_s}{(M+2)^2+M(M+2)t_s}. \end{cases} \tag{16}$$

In addition,

$$\frac{\partial\lambda}{\partial\sigma_u}<0, \qquad \frac{\partial\lambda}{\partial\sigma_v}>0. \tag{17}$$

(3)

$$\frac{\partial\gamma_i}{\partial M}\begin{cases} <0 & if \quad k>\dfrac{Mt_s-(M+2)}{2(M+2)+2(M+1)t_s}, \\ \geq 0 & if \quad \frac{-(M+2)^2+M^2t_s}{2(M+2)^2+2M(M+2)t_s}<k\leq\frac{Mt_s-(M+2)}{2(M+2)+2(M+1)t_s}. \end{cases} \tag{18}$$

$$\frac{\partial\gamma_i}{\partial t_s}\begin{cases} < 0 & if \quad k > \dfrac{M}{2(M+2)}, \\ \geq 0 & if \quad \frac{-(M+2)^2+M^2 t_s}{2(M+2)^2+2M(M+2)t_s} < k \leq \frac{M}{2(M+2)}. \end{cases} \tag{19}$$

In addition,

$$\frac{\partial\gamma_i}{\partial k} < 0, \qquad \frac{\partial\gamma_i}{\partial\sigma_u} > 0, \qquad \frac{\partial\gamma_i}{\partial\sigma_v} < 0. \tag{20}$$

From the above proposition, we have the following: (1) In the equilibrium, α and γ_i are all decreasing functions of k, which means that the lower the price is, the more aggressively the insider and outsiders trade on their private information. (2) The adverse selection, measured by λ, is decreasing with k when k is large enough ($k \geq \frac{2Mt_s+2M^2t_s}{(M+2)^2+M(M+2)t_s}$). (3) When $k > \frac{M}{M+2}$, α is a decreasing function of M, and when $k > \frac{M}{2(M+2)}$, α is a decreasing function of t_s.

Moreover, the insider's trading intensity on his/her private information and the insider and outsiders' trading intensities on the shared information all vary negatively with price derivation, variance of the asset's value, and variance of the noise volume. When the derivation is big enough, the insider's trading intensity on his/her private information and the insider and outsiders' trading intensities on the shared information all vary negatively with the number of informed outsiders and variance of shared information. The market depth varies positively with variance of the asset's value and negatively with variance of the noise volume. Additionally, when the deviation is big enough, the market depth varies positively with the number of informed outsiders and variance of shared information and negatively with the price deviation.

As the limit when t_s and k approach zero, we reach the model in Kyle (1985) with no outsiders. Note that as t_s approaches 0, the shared information and order from the outsiders simultaneously disappear. Our results then reduce to Theorem 1 of Kyle (1985).

4.2. Information revelation

To obtain a measure of the informativeness of price, we define

$$\Sigma_1 = var\{\tilde{v}|\tilde{p}\},$$

which is a measure of the residual information after information is incorporated into the price.

Let $I(t_s, t_\epsilon) = var(\tilde{v}) - \Sigma_1$. It, therefore, measures how much information has been incorporated into the equilibrium price. We have the following proposition.

Proposition 2. *In the equilibrium,*

$$I(t_s, k) = \frac{M + 2 + Mt_s}{2(M + 2)(1 + k)}\sigma_v^2.$$

In particular, if there is no bias, that is, $k = 0$,

$$I(t_s, 0) = \frac{M + 2 + Mt_s}{2(M + 2)}\sigma_v^2.$$

If there is no perfect shared information, that is, $t_s = 0$,

$$I(0, k) = \frac{\sigma_v^2}{2(1 + k)}.$$

Additionally,

$$I(t_s, k) \begin{cases} \leq I(t_s, 0), & \text{if } k \geq 0, \\ > I(t_s, 0), & \text{if } \frac{-(M+2)^2 + M^2 t_s}{2(M+2)^2 + 2M(M+2)t_s} < k < 0, \end{cases}$$

and $I(t_s, k) > I(0, k)$ for all $k > \frac{-(M+2)^2 + M^2 t_s}{2(M+2)^2 + 2M(M+2)t_s}$.

Proof. See the appendix. ▮

This proposition tells us that when t_s is fixed, $I(t_s, k)$ is a decreasing function of k. This means that the lower the price is, the more information is revealed by the equilibrium price. This proposition also tells us that the existence of shared information and presence of outsiders all lead to more informativeness of the price. The bigger the size of the outsiders and/or variance of shared information is, the more information is revealed by the equilibrium price.

4.3. Profits of the insider and outsiders

Next, we consider the insider and outsiders' equilibrium profits and noise traders' expected loss. The results are given in the following two propositions.

Proposition 3. *In the equilibrium, the expected profit conditional on the information of the insider is*

$$E[\tilde{\pi} | \tilde{v} = v, \tilde{s} = s] = \frac{\sigma_u[(-1 - 2k)(M + 2)p_0 + (M + 2)v - Ms]^2}{2\sigma_v(M + 2)\sqrt{(M + 2)^2(1 + 2k) + (2M^2k + 4Mk - M^2)t_s}}.$$

Additionally, the expected profit conditional on the information of outsider i $(i = 1, \cdots, M)$ *is*

$$E[\tilde{\pi}_i|\tilde{s} = s] = \frac{2\sigma_u\left[-(M+2)kp_0 + s\right]^2}{\sigma_v(M+2)\sqrt{(M+2)^2(1+2k) + (2M^2k + 4Mk - M^2)t_s}}.$$

Their ex-ante expected profits are, respectively,

$$E[\tilde{\pi}] = \frac{\sigma_u[4k^2(M+2)^2p_0^2 + (M+2)^2(1-t_s)\sigma_v^2 + 4t_s\sigma_v^2]}{2\sigma_v(M+2)\sqrt{(M+2)^2(1+2k) + (2M^2k + 4Mk - M^2)t_s}},$$

and

$$E[\tilde{\pi}_i] = \frac{2\sigma_u\left[(M+2)^2k^2p_0^2 + t_s\sigma_v^2\right]}{\sigma_v(M+2)\sqrt{(M+2)^2(1+2k) + (2M^2k + 4Mk - M^2)t_s}}.$$

The ex-ante expected loss of noise traders is

$$E[\tilde{L}] = \lambda(1+k)\sigma_u^2 = \frac{\sigma_v\sigma_u}{2(M+2)}\sqrt{(M+2)^2(1+2k) + (2M^2k + 4Mk - M^2)t_s}.$$

The expected profit of market makers is

$$E[\pi_M] = \frac{k\sigma_u\left[-2k(M+1)(M+2)p_0^2 + (M+2)\sigma_v^2 + Mt_s\sigma_v^2\right]}{\sigma_v\sqrt{(M+2)^2(1+2k) + (2M^2k + 4Mk - M^2)t_s}}.$$

Proof. See the appendix. ▮

As to the comparative statics on the expected profits and loss, we have the following proposition.[4]

Corollary 1. *The ex-ante expected profits of the insider,* $E[\tilde{\pi}]$, *and the outsiders,* $E[\tilde{\pi}_i](i = 1, 2 \cdots, M)$, *are all decreasing functions of* k. *On the other hand, the ex-ante expected loss of noise traders and the ex-ante expected profits of market makers are all increasing functions of* k.

From the above corollary, we learn that the insider and outsiders all prefer the price to be lower than the semi-strong efficient condition. However, noise traders will lose less money when the trading happens at a price lower than the semi-strong efficient price.

[4]For computational convenience, we only give the analysis for the case of $p_0 = 0$.

5. CONCLUSION

We investigate Kyle's (1985) extended model with the setting of private and shared information and the semi-strong inefficient condition. Our analysis suggests that both an increasing number of informed outsiders and a price lower than the semi-strong efficient price result in more effectiveness of the equilibrium price. Moreover, both the insider and outsiders prefer the price to be lower than the semi-strong efficient condition, whereas market makers prefer a higher price than the semi-strong efficient case.

APPENDIX: PROOF

We first state a well known regression result that will used later.

Lemma 1. Let X_1 and X_2 have joint normal distribution, $\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N(\mu, \Sigma)$ with $\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$, $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$. Then the random variable X_1 conditional on X_2 has a normal distribution, and

$$E[X_1|X_2] = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(X_2 - \mu_2), Var(X_1|X_2) = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}.$$

Proof of theorem 1. We conjecture that the linear equilibrium is given by

$$\tilde{x} = X(\tilde{v}, \tilde{s}) = a + \alpha\tilde{v} + \beta\tilde{s}, \tag{A.1}$$

$$\tilde{y}_i = Y_i(\tilde{s}) = b_i + \gamma_i\tilde{s}, \; i = 1, \cdots, M, \tag{A.2}$$

$$\tilde{p} = P(\tilde{y}) = (1 + k)(c + \lambda\tilde{y}), \tag{A.3}$$

where the parameters a, b_i, c, α, β, γ_i and λ are constants that need to be determined. We will identify the parameters and verify the conjecture.

Since $\tilde{x}$ is a measurable function of $\tilde{v}$ and $\tilde{s}$, the insider's expected profit conditional on his information is

$$\begin{aligned} & E[(\tilde{v} - \tilde{p})x|\tilde{v}, \tilde{s}] \\ =& E\{[\tilde{v} - (1 + k)c - (1 + k)\lambda(x + \tilde{y}_1 + \cdots + \tilde{y}_M + \tilde{u})]x|\tilde{v}, \tilde{s}\} \\ =& E\{[\tilde{v} - (1 + k)c - (1 + k)\lambda x - (1 + k)\lambda\sum_{i=1}^{M} b_i - (1 + k)\lambda\sum_{i=1}^{M}\gamma_i\tilde{s}]x|\tilde{v}, \tilde{s}\} \\ =& x[\tilde{v} - (1 + k)c - \lambda(1 + k)x - \lambda(1 + k)\sum_{i=1}^{M} b_i - \lambda(1 + k)(\sum_{i=1}^{M}\gamma_i)\tilde{s}]. \end{aligned} \tag{A.4}$$

By the first order condition, we have

$$\tilde{x} = -\frac{c}{2\lambda} - \frac{1}{2}\sum_{i=1}^{M} b_i + \frac{1}{2\lambda(1+k)}\tilde{v} - \frac{1}{2}(\sum_{i=1}^{M}\gamma_i)\tilde{s} \tag{A.5}$$

and the second order condition is $\lambda(1+k) > 0$. Comparing (A.5) and (A.1), we have

$$\begin{cases} a = -\frac{c}{2\lambda} - \frac{1}{2}\sum_{i=1}^{M} b_i, \\ \alpha = \frac{1}{2\lambda(1+k)}, \\ \beta = -\frac{1}{2}\left(\sum_{i=1}^{M}\gamma_i\right). \end{cases} \tag{A.6}$$

Since $\tilde{y}_i$ is a measurable function of $\tilde{s}$, the outsider i's expected profit conditional on his information is

$$\begin{aligned} &E[(\tilde{v}-\tilde{p})y_i|\tilde{s}] \\ =&E\{[\tilde{v}-(1+k)c-\lambda(1+k)(\tilde{x}+y_i+\sum_{j\neq i}\tilde{y}_j+\tilde{u})]y_i|\tilde{s}\} \\ =&y_i\{[1-\lambda(1+k)\alpha]p_0-(1+k)c-\lambda(1+k)a+[1-\lambda(1+k)\alpha]\tilde{s}-\lambda\beta(1+k)\tilde{s}- \\ &\lambda(1+k)\sum_{j\neq i}\tilde{y}_j-\lambda(1+k)y_i\}. \end{aligned} \tag{A.7}$$

By the first order condition, we have

$$\tilde{y}_i = \frac{1}{2\lambda(1+k)}\{[1-\lambda(1+k)\alpha]p_0-(1+k)c-\lambda(1+k)a+[1-\lambda(1+k)\beta-\lambda(1+k)\alpha]\tilde{s}\}-\frac{1}{2}\sum_{j\neq i}\tilde{y}_j,\ i=1,\dots,M. \tag{A.8}$$

It is easy to see that this system of linear equations of $\tilde{y}_1,\dots,\tilde{y}_M$ has a unique solution.

By the symmetry, we immediately get that

$$\begin{aligned} &\tilde{y}_1 = \tilde{y}_2 = \cdots = \tilde{y}_M \\ =&\frac{1}{(M+1)(1+k)\lambda}\left\{[1-\lambda(1+k)\alpha]p_0-(1+k)c-\lambda(1+k)a+[1-\lambda(1+k)\beta-\lambda(1+k)\alpha]\tilde{s}\right\}. \end{aligned} \tag{A.9}$$

And the second order condition is $\lambda(1+k) > 0$.

Comparing (A.9) and (A.2), we have

$$\begin{cases} b_i = \frac{[1-\lambda(1+k)\alpha]p_0-(1+k)c-\lambda(1+k)a}{(M+1)\lambda(1+k)}, \\ \gamma_i = \frac{1-\lambda\alpha(1+k)-\lambda\beta(1+k)}{(M+1)\lambda(1+k)}. \end{cases} \tag{A.10}$$

for all $i = 1, \cdots, M$.

Now by the semi-strong efficient condition of the market, we have

$$P(\tilde{y}) = (1+k)E[\tilde{v}|\tilde{x} + \tilde{y}_1 + \cdots + \tilde{y}_M + \tilde{u}] \tag{A.11}$$

From $(A.1)$ and $(A.2)$, we know that

$$\tilde{x} + \tilde{y}_1 + \cdots + \tilde{y}_M + \tilde{u} = a + Mb_i + \alpha\tilde{\zeta} + (\alpha + \beta + M\gamma_i)\tilde{s} + \tilde{u}.$$

Therefore

$$\begin{pmatrix} \tilde{v} \\ \tilde{x} + \tilde{y}_1 + \cdots + \tilde{y}_M + \tilde{u} \end{pmatrix} = \begin{pmatrix} \tilde{v} \\ a + Mb_i + \alpha\tilde{\zeta} + (\alpha + \beta + M\gamma_i)\tilde{s} + \tilde{u} \end{pmatrix}$$
$$\sim N\left(\begin{pmatrix} p_0 \\ a + Mb_i + \alpha p_0 \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right), \tag{A.12}$$

where

$$\Sigma_{11} = \sigma_v^2,$$

$$\Sigma_{12} = \Sigma_{21} = [\alpha(1-t_s) + (\alpha + \beta + M\gamma_i)t_s]\sigma_v^2,$$

$$\Sigma_{22} = \alpha^2(1-t_s)\sigma_v^2 + (\alpha + \beta + M\gamma_i)^2 t_s \sigma_v^2 + \sigma_u^2,$$

By Lemma 1, we have

$$E\left(\tilde{v}|\tilde{x} + \tilde{y}_1 + \cdots + \tilde{y}_M + \tilde{u}\right) = p_0 + \frac{[\alpha(1-t_s) + (\alpha + \beta + M\gamma_i)t_s]\sigma_v^2}{\alpha^2(1-t_s)\sigma_v^2 + (\alpha + \beta + M\gamma_i)^2 t_s \sigma_v^2 + \sigma_u^2}(\tilde{y} - a - Mb_i - \alpha p_0) \tag{A.13}$$

So we get

$$\begin{cases} c = p_0 - \lambda a - \lambda M b_i - \lambda\alpha p_0, \\ \lambda = \frac{[\alpha(1-t_s) + (\alpha+\beta+M\gamma_i)t_s]\sigma_v^2}{\alpha^2(1-t_s)\sigma_v^2 + (\alpha+\beta+M\gamma_i)^2 t_s\sigma_v^2 + \sigma_u^2} \end{cases} \tag{A.14}$$

From (A.6) and (A.10) we have

$$\alpha = \frac{1}{2\lambda(1+k)}, \qquad \gamma_i = \frac{1}{\lambda(1+k)(M+2)}, \qquad \beta = -\frac{M}{2\lambda(1+k)(M+2)}. \tag{A.15}$$

(A.6), (A.10) and (A.14) imply

$$a = -\frac{1+2k}{2\lambda(1+k)}p_0, \tag{A.16}$$

$$b_i = -\frac{k}{\lambda(1+k)}p_0, \tag{A.17}$$

$$c = \frac{1+(M+2)k}{1+k} p_0, \tag{A.18}$$

for all $i = 1, \cdots, M$.

Substituting (A.15) into (A.14),

$$\lambda^2 (1+k)^2 = \frac{\sigma_v^2}{\sigma_u^2} \left\{ \frac{1+2k}{4}(1-t_s) + \frac{(M+1)(1+Mk+2k)}{(M+2)^2} t_s \right\}. \tag{A.19}$$

The above equation has the solution only when

$$\frac{1+2k}{4}(1-t_s) + \frac{(M+1)(1+Mk+2k)}{(M+2)^2} t_s > 0$$

i.e.,

$$k > \frac{-(M+2)^2 + M^2 t_s}{2(M+2)^2 + 2M(M+2)t_s}. \tag{A.20}$$

Taking into account of the second order condition of the insider's optimization problem, we get

$$\begin{aligned} \lambda =& \frac{\sigma_v}{(1+k)\sigma_u} \sqrt{\frac{1+2k}{4}(1-t_s) + \frac{(M+1)(1+Mk+2k)}{(M+2)^2} t_s} \\ =& \frac{\sigma_v}{2(1+k)\sigma_u (M+2)} \sqrt{(M+2)^2(1+2k) + (2M^2k + 4Mk - M^2)t_s}. \end{aligned} \tag{A.21}$$

Now we complete the proof.

Proof of Proposition 1. Since

$$\alpha = \frac{1}{2\lambda(1+k)} = \frac{\sigma_u (M+2)}{\sigma_v} \frac{1}{\sqrt{(M+2)^2(1+2k) + (2M^2k + 4Mk - M^2)t_s}},$$

$$\lambda = \frac{\sigma_v}{2(1+k)\sigma_u (M+2)} \sqrt{(M+2)^2(1+2k) + (2M^2k + 4Mk - M^2)t_s},$$

$$\gamma_i = \frac{2\sigma_u}{\sigma_v \sqrt{(M+2)^2(1+2k) + (2kM^2 - M^2 + 4Mk)t_s}},$$

it is easy to know that

$$\frac{\partial \alpha}{\partial k} < 0, \qquad \frac{\partial \alpha}{\partial \sigma_v} < 0, \qquad \frac{\partial \alpha}{\partial \sigma_u} > 0,$$

$$\frac{\partial\lambda}{\partial\sigma_v} > 0, \qquad \frac{\partial\lambda}{\partial\sigma_u} > 0$$

$$\frac{\partial\gamma_i}{\partial k} < 0, \qquad \frac{\partial\gamma_i}{\partial\sigma_v} < 0, \qquad \frac{\partial\gamma_i}{\partial\sigma_u} > 0,$$

and if $2M^2k + 4Mk - M^2 > 0$, i.e., $k > \frac{M}{2(M+2)}$,

$$\frac{\partial\alpha}{\partial t_s} < 0, \qquad \frac{\partial\lambda}{\partial t_s} > 0, \qquad \frac{\partial\gamma_i}{\partial t_s} < 0.$$

Also,

$$\frac{\partial\alpha}{\partial M} = \frac{\sigma_u(-2kM - 4k + 2M)t_s}{\sigma_v[\sqrt{(M+2)^2(1+2k) + (2M^2k + 4Mk - M^2)t_s}]^3},$$

$$\frac{\partial\lambda}{\partial M} = \frac{\sigma_v(2kM + 4k - 2M)t_s}{2\sigma_u(1+k)(M+2)^2\sqrt{(M+2)^2(1+2k) + (2M^2k + 4Mk - M^2)t_s}},$$

$$\frac{\partial\gamma_i}{\partial M} = \frac{2\sigma_u[2(M+2)(1+2k) + (4kM + 4k - 2M)t_s}{\sigma_v[(M+2)^2(1+2k) + (2M^2k + 4Mk - M^2)t_s]}.$$

From $(A.21)$, it is easy to know that

$$\frac{\partial\lambda}{\partial k} = \frac{\sigma_v[-k(M+2)^2 + (2M^2 + 2M - kM^2 - 2Mk)t_s]}{2\sigma_u(1+k)^2(M+2)\sqrt{(M+2)^2(1+2k) + (2M^2k + 4Mk - M^2)t_s}}. \tag{A.22}$$

so $\frac{\partial\lambda}{\partial k} > 0$ if $-k(M+2)^2 + (2M^2 + 2M - kM^2 - 2Mk)t_s > 0$, i.e., $k < \frac{2Mt_s + 2M^2t_s}{(M+2)^2 + M(M+2)t_s}$. Now the rest of the proof is straightforward.

Proof of Proposition 2. Since

$$\tilde{p} = (1+k)[c + \lambda a + \lambda M b_i + \lambda\alpha\tilde{v} + (\lambda\beta + \lambda M\gamma_i)\tilde{s} + \lambda\tilde{u}],$$

by Lemma 1, we have

$$var(\tilde{v}|\tilde{p}) = \sigma_v^2 - \frac{(\alpha + \beta t_s + M\gamma_i t_s)^2\sigma_v^4}{\alpha^2(1-t_s)\sigma_v^2 + (\beta + \alpha + M\gamma_i)^2 t_s\sigma_v^2 + \sigma_u^2}.$$

Substitute the expression of $\beta, \theta, \delta_i, \gamma_i, \eta$ and λ in Theorem 1 into the above equation, we have

$$I(t_s, k) = \frac{(M + 2 - Mt_s + 2Mt_s)^2\sigma_v^4}{(M+2)^2(1-t_s)\sigma_v^2 + 4(M+1)^2 t_s\sigma_v^2 + 4\lambda^2(1+k)^2(M+2)^2\sigma_u^2}.$$

Substitute the expression of λ into the above, we have

$$I(t_s,k)=\frac{(M+2+Mt_s)\sigma_v^2}{2(M+2)(1+k)}.$$

$$\begin{aligned}I(t_s,k)-I(0,k)=&\frac{(M+2+Mt_s)\sigma_v^2}{2(M+2)(1+k)}-\frac{\sigma_v^2}{2(1+k)}\\=&\frac{Mt_s}{2(1+k)(M+2)}\sigma_v^2>0.\end{aligned}$$

so $I(t_s,k)>I(0,k)$.

$$\begin{aligned}I(t_s,k)-I(t_s,0)=&\frac{(M+2+Mt_s)\sigma_v^2}{2(M+2)(1+k)}-\frac{(M+2+Mt_s)\sigma_v^2}{2(M+2)}\\=&-\frac{(M+2+Mt_s)k}{2(1+k)(M+2)}\sigma_v^2.\end{aligned}$$

Now the rest of the proof is straightforward.

Proof of Proposition 3. From Theorem 1 and Equation (A.4), the condition expected profit of the insider is

$$\begin{aligned}&E[\tilde{\pi}|\tilde{v}=v,\tilde{s}=s]=E[(\tilde{v}-\tilde{p})\tilde{x}|\tilde{v}=v,\tilde{s}=s]\\=&[a+\alpha v+\beta s][v-(1+k)c-\lambda(1+k)a-\lambda(1+k)\alpha v-\lambda(1+k)\beta s-\lambda(1+k)Mb_i-\lambda(1+k)M\gamma_i s]\\=&\left[\frac{-1-2k}{2\lambda(1+k)}p_0+\frac{1}{2\lambda(1+k)}v-\frac{M}{2\lambda(1+k)(M+2)}s\right]\\&\left[v-(1+(M+2)k)p_0+\frac{1+2k}{2}p_0-\frac{1}{2}v+\frac{M}{2(M+2)}s+Mkp_0-\frac{M}{M+2}s)\right]\\=&\frac{1}{4\lambda(1+k)(M+2)^2}\left[(-1-2k)(M+2)p_0+(M+2)v-Ms\right]^2\\=&\frac{\sigma_u[(-1-2k)(M+2)p_0+(M+2)v-Ms]^2}{2\sigma_v(M+2)\sqrt{(M+2)^2(1+2k)+(2M^2k+4Mk-M^2)t_s}}.\end{aligned}$$

Hence

$$\begin{aligned}E[\tilde{\pi}]=&\frac{\sigma_u[(1+2k)^2(M+2)^2p_0^2+(M+2)^2((1-t_s)\sigma_v^2+p_0^2)+4t_s\sigma_v^2+2(-1-2k)(M+2)^2p_0^2]}{2\sigma_v(M+2)\sqrt{(M+2)^2(1+2k)+(2M^2k+4Mk-M^2)t_s}}\\=&\frac{\sigma_u[4k^2(M+2)^2p_0^2+(M+2)^2(1-t_s)\sigma_v^2+4t_s\sigma_v^2]}{2\sigma_v(M+2)\sqrt{(M+2)^2(1+2k)+(2M^2k+4Mk-M^2)t_s}}.\end{aligned}$$

From Theorem 1 and Equation (A.7), the conditional expected profit of outsider i is

$$\begin{aligned}
&E[\tilde{\pi}_i|\tilde{s}=s] = E[(\tilde{v}-\tilde{p})\tilde{y}_i|\tilde{s}=s]\\
=&[b_i+\gamma_i s]\{[1-\lambda(1+k)\alpha]p_0+[1-\lambda(1+k)\alpha]s-(1+k)c-\lambda(1+k)a-\lambda(1+k)\beta s\\
&-\lambda(1+k)M\gamma_i s-\lambda(1+k)Mb_i\}\\
=&\frac{1}{\lambda(1+k)(M+2)^2}\left[-(M+2)kp_0+s\right]^2\\
=&\frac{2\sigma_u\left[-(M+2)kp_0+s\right]^2}{\sigma_v(M+2)\sqrt{(M+2)^2(1+2k)+(2M^2k+4Mk-M^2)t_s}}.
\end{aligned}$$

Therefore the outsiders' ex ante expected profit is

$$E[\tilde{\pi}_i] = \frac{2\sigma_u\left[(M+2)^2k^2p_0^2+t_s\sigma_v^2\right]}{\sigma_v(M+2)\sqrt{(M+2)^2(1+2k)+(2M^2k+4Mk-M^2)t_s}}.$$

The loss of noise traders is

$$\begin{aligned}
\tilde{L} =&(\tilde{p}-\tilde{v})\tilde{u}\\
=&\tilde{u}[(1+k)c+\lambda(1+k)a+\lambda(1+k)\alpha\tilde{v}+\lambda(1+k)\beta\tilde{s}+\lambda(1+k)Mb_i+\lambda(1+k)M\gamma_i\tilde{s}-\tilde{v}]+\lambda(1+k)\tilde{u}^2.
\end{aligned}$$

Their expected loss is therefore

$$E[\tilde{L}] = \lambda(1+k)\sigma_u^2 = \frac{\sigma_v\sigma_u}{2(M+2)}\sqrt{(M+2)^2(1+2k)+(2M^2k+4Mk-M^2)t_s}.$$

The expected profit of market makers is

$$E[\pi_M] = E\{(-\tilde{y})[\tilde{v}-(1+k)E(\tilde{v}|\tilde{y})]\} = kE[\tilde{y}\tilde{v}] = kE\{\tilde{v}[a_M b_i+\alpha\tilde{\zeta}+(\alpha+\beta+M\gamma_i)\tilde{s}+\tilde{u}]\}.$$

Substitute the expressions of a, b_i, α, β and γ_i given by Theorem 1, we can get

$$E[\pi_M] = \frac{k\sigma_u\left[-2k(M+1)(M+2)p_0^2+(M+2)\sigma_v^2+Mt_s\sigma_v^2\right]}{\sigma_v\sqrt{(M+2)^2(1+2k)+(2M^2k+4Mk-M^2)t_s}}.$$

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