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Adverse Selection and Moral Hazards Reduction in Corporate Financing: A Mechanism Design Model for PLS Contracts

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In this paper, we apply game theory to corporate financing using profit and loss sharing (PLS) contracts. We employ mechanism design theory using two agents, a bank and a corporation which seeks financing through PLS mode. We seek to find the usefulness of mechanism design in helping the bank separating low type from high type corporations by designing two bundles of contract with each contract directed in a compatible way towards the appropriate type of corporation. We found theoretical as well as simulation evidence that our model helps in minimizing asymmetric information in the form of adverse selection by forcing the corporation to reveal its type. The model also helps in reducing asymmetric information in the form of moral hazards. This is achieved by having the selected high type corporation select a high type contract using a moral hazard premium as an incentive.

Key Words: Sharing ratio; Adverse selection; Assymetric information; Moral hazards; Moral hazard premium (MHP); PLS contracts.
JEL Classification Numbers: C700, G32, D81, D82.

1. INTRODUCTION

One important part in decision making in a collective setting is that individuals' types and preferences are not observable. Therefore, mechanisms have to be inroduced to elicit individuals into revealing this type of information. How these information can be revealed and how decision makers can respond to individuals' preferences based on such information is referred to as mechanism design (Èihák et al., 2008). The challenge of a mechanism designer is that the outcome of self-interested parties should be acceptable to all. In recognition of the importance of mechanism design,

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The Royal Swedish Academy of Sciences has awarded the 2007 Nobel Prize in economic sciences to three economists Leonid Hurwicz, Eric Maskin, and Roger Myerson. Mechanism design started with the work of Hurwicz (1960).

In agreement with the works of Friedrich von Hayek (1945), Hurwicz confirmed that the failure of central planning was a result of information dispersion and assymmetries among economic agents. However, he argued that the lack of incentives for economic agents prohibited them from sharing truthful information with others. To overcome this problem Hurwicz (1972) introduced the term incentive-compatibility which stresses on Information sharing. Yet, this mechanism does not guarantee optimality as the existence of assymmetric information precludes Pareto efficiency (Èihák et al., 2008). However this mechanism can be close to optimality than the case without incentive compatibility (Èihák et al., 2008).

Another contribution to mechanism design, is the notion of "Revelation principle" introduced by Roger Myerson. It simplifies the calculation of the most efficient rules to get agent reveal their true information. Major extentions of works about the revelation principle include Gibbard (1973), Dasgupta et al. (1979) and Myerson (1979).

When it comes to Eric Maskin's contribution, he has coined the "Implementation Theory" (Maskin, 1999) which became a radical part in mechanism design. It states when mechanisms can be designed that produce incentive efficient equilibria. The idea is that under Myerson's and others work many solutions can arise with different equilibria. Yet not all these equilibria are in fact implementable. The importance of mechanism design in tackling information assymmetries can be applied to many fields. One such field is Profit and Loss sharing contracts (PLS) referred to in Islamic finance as Musharakah. It is namely a partnership contract between a capital and labor providers. The sharing of profits is in accordance with a pr-determined ratio (Rammal, 2004) while Losses are limited to partners contributions (Usmani, 2002).

Mechanism design is mostely needed in Islamic Musharakah than in conventional PLS contracts due to the following reasons:

• Fixed compensations are allowed on conventional system while they are prohibited in Islamic Musharakah. The Financier in the later mode might lose if the project fails.

• The financed agent is usually the provider of a lower share of capital. He/she has an incentive to behave in an opportunistic manner.

• In conventional system, the financier might seek protection against losses. this is not permissible under Islamic musharakh (Usmani, 2002).

The rest of the papers will be as follows:

Section 2 reviews the literature. Section 3 elaborates the model. Section 4 explains the methods used. Section 5 illustrates the results. Section 6 presents a numerical application as well as a computer aided simulation. section 7 is devoted to the discussion of the results. Finally, Section 9 presents a summary and suggests future extensions.

2. LITERATURE REVIEW

To reduce assymmetric information that an agent holds against a financier many dissipative signals can by exploited.

One such signal is collateral that efficient agents need to use to signal their efficient type. This is consistent with works of Berger et al. (2011) and Karim (2002). While this method is allowed in a conventional system, it is prohibited in Islamic jurisprudence (Shariah law). The recourse to a warranty is allowed only if there is a proof of negligence or violation of the terms of the PLS contracts [1].

Benchmarking on collateral, efficient managers might sign for low job protection (Subramanian and Sheikh, 2002). This is in agreement with other research as in (Subramanian and Sheikh, 2002). Yet, signing for low job protection is considered unfair to the entrepreneur since the project failures can be due to some uncontrollable factors l (ELFakir and Tkiouat, 2015a), (ElFakir and Tkiouat, 2015b).

Due diligence can be used to reduce moral hazards, in the form of misreporting, in Musharakah. However, the extent of it is more in Musharakah than in conventional PLS Al Suwailem (2006).

The Entrepreneur's participation in the project's capital might be used to reduce information assymmetries in Musharakah contracts (Karim, 2002). In agreement with this argument, we have allowed the agent to participate in the project capital in our model.

In dealing with moral hazard, in the form of low effort, Nabi (2013) suggested that the entrepreneur should participate with a minimum capital contribution coupled with a minimum profit sharing ration. In our model we propose the usage of two types of contracts (High and low) with each type directed toward its compatible type of entrepreneurs.

One research suggested that moral hazards can be reduced under Mudaraba[2] but cannot be solved under Musharakah (Yousfi, 2013). We can criticize this argument in the following way. First under Mudaraba, the financier is the sole provider of capital and subsequently supports all types

[1] Adoption of AAOIFI Shariah Standard No. 12. Clause 3/1/4/1

[2] A form of business where the financier provides capital while the entrepreneur provides work.

of risks. On the other hand, under losses are shared. Our reasoning is inconsistent with the findings of Nabi (2013) and Innes (1990).

In a previous paper (ELFakir and Tkiouat, (2015a) of ours we have proposed an incentive mechanism to reduce assymmetric information. This mechanism results in higher social value and more entrepreneurial negotiation power in terms of the profit sharing ratio. The model however does not provide for two contracts type as the current model does.

In order to assess the viability PLS contracts versus other modes of financing such as ROSCA and debt-finance we have proposed in a previous paper a new model called ROMCA (ELFakir and Tkiouat, 2016). Our Simulation results show that our rotating Musharakah model, ROMCA, prevails against debt finance when it comes to employment generation, wealth creation, and consumption. It becomes even dominant under cases of adverse random shocks with low market conditions and prevailed in cases of moral hazards (ELFakir and Tkiouat, 2016).

In dealing with adverse selection and moral hazards, we have provided some series of publications using two contracts. The purpose is to allow for agents type separation. In the rst paper, we suggested using two types of contracts: one is eort based and the second is output based. Theoretical evidence showed that an eort based contract can give higher compensation to the agent as this contract oers a lower sharing ratio to the nancier (ELFakir and Tkiouat, 2015b). This result emphasis two important Islamic concepts. First it emphasizes the sentiment of altruism which the nancier shows by taking a smaller prot sharing ratio. Second it emphasizes the sentiment of positive reciprocity which the agent exhibits by providing high eort.

In the second paper, we tried to reduce the adverse selection with respect to Mudaraba using a model of two contracts combined with adverse selection index for each contract. We have manged to develop three types of indexes that can help nancial institutions in their agent selection process (ELFakir and Tkiouat, 2016a).

In the third paper, we tried to use a two contract concept in a game theoretical approach under incomplete information. Menu contracting was found not to be always the optimal option for moral hazard reduction ELFakir and Tkiouat, (2016b).

3. THE MODEL

Our Model involves a Bank and a corporation engaging in a Musharakah contract wit a required funding F. Due to asymmetric information, the bank can only provide partial financing and, therefore, need to decide on the appropriate portion β of F to be delivered to the corporation. The project provides a concave return $V_{(\beta)}$ with respect to the corporation capital contribution β. The return function is common knowledge adjusted with a productivity Θ which is private to the corporation and unknown to the bank. The bank has a monitoring cost $C_{(\beta)}$ which is a function of its contribution in the project such that: $C_{(\beta)} = c.F.\beta$ (Where $0 < c < 1$ and $0 < \beta < 1$).

The corporation can be of two types: High with probability $\overline{P}$ or Low with probability $\underline{P}$. A high (Low) type corporation have a productivity factor $\overline{\Theta}$ ($\underline{\Theta}$) and results in a high (Low) output than a low (High) type and therefore merit a high (Low) financing $\overline{\beta}$ ($\underline{\beta}$). We note as well that $\overline{\Theta} > \underline{\Theta}$.

The payoff of the project when undertaken by the high type corporation is $\overline{\Theta}.V_{(\overline{\beta})}$ while it is $\underline{\Theta}.V_{(\underline{\beta})}$ if undertaken by the low type corporation.

3.1. The Model under the Symmetric case

In this case, the bank knows the type of the corporation. The bank shares the profit according to a predetermined sharing ratio α and losses according to β. The corporation gets under the symmetric case:

$$U_{corp/sym} = (1-\alpha).\Theta V_{(\beta)} - (1-\beta).F$$

which can be rewritten as:

$$U_{corp/sym} = \Theta V_{(\beta)} - [\alpha.\Theta V_{(\beta)} + (1-\beta).F] \tag{1}$$

We can put the second part of the equation:

$$T_{(\beta)} = [\alpha.\Theta V_{(\beta)} + (1-\beta).F] \tag{2}$$

This part represents the transfer of the project proceeds from the corporation to the bank. So the bank gives $\beta.F$ to the corporation and receives back $T_{(\beta)}$.

Since the bank knows the type of the corporation, it can set β in such a way that the corporation just breaks even. i.e

$$\Theta V_{(\beta)} - [\alpha.\Theta V_{(\beta)} + (1-\beta).F] = 0$$

Or alternatively

$$\Theta V_{(\beta)} = T_{(\beta)}$$

So automatically $T_{(\beta)}$ is also concave. The banks profit under the symmetric case is then:

$$U_{bank/sym} = T_{(\beta)} - F - c.B.F$$

Or alternatively:

$$U_{bank/sym} = T_{(\beta)} - F.(1 + c.B) \tag{3}$$

This is also a concave function which is a reasonable assumption for a risk averse financier. We can then Maximize the bank's profit at:

$$\Theta V'_{(\beta)} = c.F \tag{4}$$

The last equation has some important implications:

1) if $\Theta = 1$ the bank will keep increasing its financing β to a level that makes its marginal profit equal to its marginal cost i.e $V'_{(\beta)} = c.F$

2) if $\Theta > 1$ the bank will keep increasing its financing β to a level that makes its marginal profit equal to its marginal cost adjusted by the productivity factor Θ i.e $V'_{(\beta)} = (c.F)/\Theta$ This means that the bank can maximize its profit at a lower level of financing if the corporation productivity is high

3) if $0 < \Theta < 1$ the bank will keep increasing its financing β to a level that makes its marginal profit equal to its marginal cost adjusted by the productivity factor Θ i.e $V'_{(\beta)} = (c.F)/\Theta$ This means that the bank can maximize its profit at a higher level of financing if the corporation productivity is low

3.2. The model under asymmetric case

In this case the bank does not know whether it deals with a low type corporation (Lcorp) or a high type corporation (Hcorp). Therefore, the payoffs to the bank from the high type contract (Hcont) and the low type contract (Lcont) are respectively:

$$U_{bank/(asym;Hcont)} = \overline{P}.[T_{(\overline{\beta}/Hcorp)} - F.(1+c.\overline{\beta})] + \underline{P}.[T_{(\overline{\beta}/Lcorp)} - F.(1+c.\overline{\beta})] \tag{5}$$

$$U_{bank/(asym;Lcont)} = \overline{P}.[T_{(\underline{\beta}/Hcorp)} - F.(1+c.\underline{\beta})] + \underline{P}.[T_{(\underline{\beta}/Lcorp)} - F.(1+c.\underline{\beta})] \tag{6}$$

Where this time:

$$T_{(\underline{\beta}/Hcorp)} = [\alpha.\underline{\Theta}V_{(\underline{\beta})} + (1 - \underline{\beta}).F] \tag{7}$$

$$T_{(\overline{\beta}/Hcorp)} = [\alpha.\overline{\Theta}V_{(\overline{\beta})} + (1 - \overline{\beta}).F] \tag{8}$$

$$T_{(\overline{\beta}/Lcorp)} = [\alpha.\underline{\Theta}V_{(\overline{\beta})} + (1 - \overline{\beta}).F] \tag{9}$$

$$T_{(\underline{\beta}/Lcorp)} = [\alpha.\underline{\Theta}V_{(\underline{\beta})} + (1 - \underline{\beta}).F] \tag{10}$$

To be able to enter the contract, the corporation needs to satisfy two types of constraints: individual rationality constraints and incentive compatibility constraints.

An individual rationality constraints means that the corporation can only enter the project if its return from the projects is greater than its reservation utility (0 in our case). So the individual rationality constraints for the low and high type corporation respectively are given as:

$$(IR_L) \qquad \underline{\Theta}V_{(\underline{\beta})} - T_{(\underline{\beta}/Lcorp)} \geq 0 \tag{11}$$

$$(IR_H) \qquad \overline{\Theta}V_{(\overline{\beta})} - T_{(\overline{\beta}/Hcorp)} \geq 0 \tag{12}$$

An incentive compatible constraints means that the type of contract is compatible with the type of the corporation that undertakes it:

$$(IC_L) \qquad \underline{\Theta}V_{(\underline{\beta})} - T_{(\underline{\beta}/Lcorp)} \geq \underline{\Theta}V_{(\overline{\beta})} - T_{(\underline{\beta}/Lcorp)} \tag{13}$$

$$(IC_H) \qquad \overline{\Theta}V_{(\overline{\beta})} - T_{(\overline{\beta}/Hcorp)} \geq \underline{\Theta}V_{(\underline{\beta})} - T_{(\underline{\beta}/Hcorp)} \tag{14}$$

In summary, the model is, then, an optimization problem taking the form:
Maximize:

$$U_{bank/(asym;Hcont)} = \overline{P}.[T_{(\overline{\beta}/Hcorp)} - F.(1+c.\overline{\beta})] + \underline{P}.[T_{(\overline{\beta}/Lcorp)} - F.(1+c.\overline{\beta})]$$

$$U_{bank/(asym;Lcont)} = \overline{P}.[T_{(\underline{\beta}/Hcorp)} - F.(1+c.\underline{\beta})] + \underline{P}.[T_{(\underline{\beta}/Lcorp)} - F.(1+c.\underline{\beta})]$$

Subject to constraints:

$$\begin{aligned}
(IR_L) &= \underline{\Theta}V_{(\underline{\beta})} - T_{(\underline{\beta}/Lcorp)} \geq 0 \\
(IR_H) &= \overline{\Theta}V_{(\overline{\beta})} - T_{(\overline{\beta}/Hcorp)} \geq 0 \\
(IC_L) &= \underline{\Theta}V_{(\underline{\beta})} - T_{(\underline{\beta}/Lcorp)} \geq \underline{\Theta}V_{(\overline{\beta})} - T_{(\underline{\beta}/Lcorp)} \\
(IC_H) &= \overline{\Theta}V_{(\overline{\beta})} - T_{(\overline{\beta}/Hcorp)} \geq \underline{\Theta}V_{(\underline{\beta})} - T_{(\underline{\beta}/Hcorp)} \\
(FC) &= 0 < \underline{\beta} < 1 \text{ and } 0 < \overline{\beta} < 1
\end{aligned}$$

Where the last is called the feasibility constraints where the contribution ratios are between 0 and 1. The bank problem is to decide on the bundle $(\underline{\beta}; T_{(\underline{\beta})})$ and the bundle $(\overline{\beta}; T_{(\overline{\beta})})$

4. METHODOLOGY

We start by deciding on which constraints are binding. To do so we provide a game theoretical approach for each type of corporation. The strategy of the bank is which bundle to offer among:

$$(\overline{\beta}; T_{(\overline{\beta}/Hcorp)}); (\overline{\beta}; T_{(\overline{\beta}/Lcorp)}); (\overline{\beta}; T_{(\overline{\beta}/Hcorp)}); (\overline{\beta}; T_{(\overline{\beta}/Lcorp)}).$$

On the other side, the strategy of the corporation is whether it should participate in the contract or not. We proceed by dealing with each type of corporation at a time:

4.1. The game between the bank and the low type company

From (7) the transfer $T_{(\underline{\beta})}$ depends on $\underline{\beta}$ and therefore the bundle ($\overline{\beta}$; $T_{(\underline{\beta})}$) represents an incompatible combination. Likewise from (6) the transfer $T_{(\overline{\beta})}$ depends on $\overline{\beta}$ and therefore the bundle ($\underline{\beta}$; $T_{(\overline{\beta})}$) represents an incompatible combination.

To make the high type corporation break even, the maximum it can get is:

$$\overline{\Theta}.V_{(\overline{\beta})} - T_{(\overline{\beta}/Hcorp)} = 0$$

We deduce automatically that:

$$\underline{\Theta}.V_{(\overline{\beta})} - T_{(\overline{\beta}/Hcorp)} < 0 \text{ Since } \underline{\Theta} < \overline{\Theta}$$

This means that the low type corporation can never choose the bundle ($\overline{\beta}; T_{(\overline{\beta}/Hcorp)}$) What is left then is the bundle ($\underline{\beta}; T_{(\underline{\beta}/Hcorp)}$) which must be an individually rational choice: i.e:

$\underline{\Theta}.V_{(\underline{\beta})} - T_{(\underline{\beta}/Lcorp)} \geq 0$. Since this is the only available choice for the corporation to participate , the bank will set the payoff of the corporation at its reservation utility given as 0. In this case IR_L must be binding.

$$\underline{\Theta}.V_{(\underline{\beta})} - T_{(\underline{\beta})} = 0 \tag{15}$$

We can then deduce the following reduced game form for both types of corporation where the corporation is to participate (P) or not participate (P) under the relevant fianncier capital contribution β.

Based on the individual rationality IR_H the bundle ($\overline{\beta}$; $T_{(\overline{\beta}/Hcorp)}$) must give at least a positive payoff for the high type corporation to be able to participate. On the other hand, it can be shown that the bundle ($\underline{\beta}$; $T_{(\underline{\beta}/Hcorp)}$) can also provide a positive payoff to the high type corporation:

Proof. We can adjust the payoff to the high type corporation from the bundle ($\underline{\beta}$; $T_{(\underline{\beta}/Hcorp)}$) by adding and substracting simultaneously$\underline{\Theta}.V_{(\underline{\beta})}$.

TABLE 1.

Payoff to the low type corporation under different bank's bundles

		Low Type Corporation	
		P	NP
Bank	$\overline{\beta}; T_{(\overline{\beta}/Hcorp)}$	a; b	0; 0
	$\underline{\beta}; T_{(\underline{\beta}/Lcorp)}$	c; d	0; 0

a; b $[T_{(\overline{\beta}/Lcorp)} - F.(1+c.\overline{\beta})]; \underline{\Theta}.V_{(\overline{\beta})} - T_{(\overline{\beta}/Lcorp)}$
c; d $[T_{(\underline{\beta}/Lcorp)} - F.(1+c.\underline{\beta})]; \underline{\Theta}.V_{(\underline{\beta})} - T_{(\underline{\beta}/Lcorp)}$

TABLE 2.

Payoff to the High type corporation under different bank's bundles

		High Type Corporation	
		P	NP
Bank	$\overline{\beta}; T_{(\overline{\beta}/Hcorp)}$	e; f	0; 0
	$\underline{\beta}; T_{(\underline{\beta}/Lcorp)}$	g; h	0; 0

e: f $[T_{(\overline{\beta}/Hcorp)} - F.(1+c.\overline{\beta})]; \overline{\Theta}.V_{(\overline{\beta})} - T_{(\overline{\beta}/Hcorp)}$
g; h $[T_{(\overline{\beta}/Hcorp)} - F.(1+c.\overline{\beta})]; \overline{\Theta}.V_{(\underline{\beta})} - T_{(\underline{\beta}/Hcorp)}$

We get:

$$\begin{aligned}\overline{\Theta}.V_{(\underline{\beta})} - T_{(\underline{\beta}/Hcorp)} =& \overline{\Theta}.V_{(\underline{\beta})} - \underline{\Theta}.V_{(\underline{\beta})} + \underline{\Theta}.V_{(\underline{\beta})} - T_{(\underline{\beta}/Hcorp)} \\ =& V_{(\underline{\beta})}(\overline{\Theta} - \underline{\Theta}) > 0\end{aligned}$$

The last result is true since from (15) we have $\underline{\Theta}.V_{(\underline{\beta})} - T_{(\underline{\beta}/Hcorp)} = 0$ and we have $(\overline{\Theta} > \underline{\Theta})$. ∎

Based on IC_H we must have:

$$\overline{\Theta}.V_{(\overline{\beta})} - T_{(\overline{\beta}/Hcorp)} \geq \overline{\Theta}.V_{(\underline{\beta})} - T_{(\underline{\beta}/Hcorp)}$$

Both bundles are individually rational for the high corporation yet only one bundle $(\overline{\beta}; T_{(\overline{\beta})})$ is incentive compatible.

Because the bank would like to minimize the rent the high type corporation can extract by undertaking a low type contract, it could reduce the payoff from $(\underline{\beta}; T_{(\underline{\beta}/Hcorp)})$ until it is just as equivalent to that of the bundle $(\overline{\beta}; T_{(\overline{\beta}/Hcorp)})$. This way the bundle $(\overline{\beta}; T_{(\overline{\beta}/Hcorp)})$ can still be individually rational and incentive compatible at the minimum rent extraction level.

To proceed we need to take the difference between the payoffs of the two bundles.

5. RESULTS

Taking the difference between the payoffs to the high type corporation from the two bundles and replacing $T_{(\overline{\beta}/Hcorp)}$ and $T_{(\underline{\beta}/Hcorp)}$ by their form in (6) and (7) gives:

$$R = [\overline{\Theta}.V_{(\underline{\beta})} - T_{(\underline{\beta}/Hcorp)}] - [\overline{\Theta}.V_{(\overline{\beta})} - T_{(\overline{\beta}/Hcorp)}]$$

Modifying we get:

$$R = V_{(\underline{\beta})}.(\overline{\Theta} - \alpha.\underline{\Theta}) - V_{(\overline{\beta})}.\overline{\Theta}.(1-\alpha) - F(\overline{\beta} - \underline{\beta}) \tag{16}$$

Where R is the amount to be reduced from the payoff of the low type contract and therefore represents an additional income to the bank.

Now we can reformulate the expected profit of the bank, the low type corporation and the high type respectively as:

$$\begin{aligned}
U_{(bank/(asym;Hcont))} &= \overline{P}.[T_{(\overline{\beta}/Hcorp)} - F.(1+c.\overline{\beta})] + \underline{P}.[T_{(\overline{\beta}/Lcorp)} - F.(1+c.\overline{\beta})] \\
U_{(bank/(asym;Lcont))} &= \overline{P}.[T_{(\underline{\beta}/Hcorp)} - F.(1+c.\underline{\beta})] + \underline{P}.[T_{(\underline{\beta}/Lcorp)} - F.(1+c.\underline{\beta})] + R \\
U_{(Hcorp/(asym;Hcont))} &= \overline{\Theta}.V_{(\overline{\beta})} - T_{(\overline{\beta}/Hcorp)} \\
U_{(Hcorp/(asym;Lcont))} &= \overline{\Theta}.V_{(\underline{\beta})} - T_{(\underline{\beta}/Hcorp)} - R \\
U_{(Lcorp/(asym;Hcont))} &= \underline{\Theta}.V_{(\overline{\beta})} - T_{(\overline{\beta}/Lcorp)} \\
U_{(Lcorp/(asym;Lcont))} &= \underline{\Theta}.V_{(\underline{\beta})} - T_{(\underline{\beta}/Lcorp)} - R
\end{aligned}$$

As we can see the amount R is added to the banks payoff and taken off from the corporation payoff when the low type contract is selected. This is done to deter the high type corporation from undertaking a low type contract.

We can now infer the following break even conditions:

For the bank to at least break even from undertaking the low type contract we must have:

$$U_{(bank/(asym;Lcont))} = \overline{P}.[T_{(\underline{\beta}/Hcorp)} - F.(1+c.\underline{\beta})] + \underline{P}.[T_{(\underline{\beta}/Lcorp)} - F.(1+c.\underline{\beta})] + R \geq 0$$

For the bank to at least break even from the high type contract we must have:

$$U_{(bank/(asym;Hcont))} = \overline{P}.[T_{(\overline{\beta}/Hcorp)} - F.(1+c.\overline{\beta})] + \underline{P}.[T_{(\overline{\beta}/Lcorp)} - F.(1+c.\overline{\beta})] \geq 0$$

For the high type corporation to at least break even from the high type contract we must have:

$$\overline{\Theta}.V_{(\overline{\beta})} - T_{(\overline{\beta}/Hcorp)} \geq 0$$

To get a clear view of our work we propose a numerical application followed by a computer aided simulation:

6. NUMERICAL APPLICATION AND SIMULATION

Consider the following output function: $V_\beta = K.(1-\beta)^{0.5}$ Where K represents the maximum output when the bank's contribution is null. This means that as the bank's contribution in the project grows, the corporation effort decreases resulting in a lower output.

The rest of the parameters are: $F = 700; c = 3\%; K = 1200; \underline{\theta} = 0.7; \overline{\theta} = 1.3; \underline{P} = 0.2; \overline{P} = 0.8; \alpha = 60\%$

We get the following:

For the high type corporation to at least break even from the high type contract we must have:

$\overline{\beta} \geq 1 - (\frac{K.\overline{\theta}.(1-\alpha)}{F})^2 \geq 20.54\%$

For the bank to at least break even from the low type contract we must have:

$U_{(bank/(asym;Lcont))} = \overline{P}.[T_{(\underline{\beta}/Hcorp)} - F.(1+c.\underline{\beta})] + \underline{P}.[T_{(\underline{\beta}/Lcorp)} - F.(1+c.\underline{\beta})] + R \geq 0$

This can be converted to a quadratic equation to get: $\underline{\beta} \leq 78.9\%$

For the bank to break even from the high type contract we must have:

$U_{(bank/(asym;Hcont))} = \overline{P}.[T_{(\overline{\beta}/Hcorp)} - F.(1+c.\overline{\beta})] + \underline{P}.[T_{(\overline{\beta}/Lcorp)} - F.(1+c.\overline{\beta})] \geq 0$

The latter equation can be turned into a quadratic equation and we can easily solve for $\overline{\beta}$. We get $\overline{\beta} \leq 66.9\%$ for the bank to break even from the hight type contract.

We can deduce that for the high type contract to be attractive to both the bank and the high type corporation, we must have:

$20.54\% \leq \overline{\beta} \leq 66.9\%$

To test our findings we use a computer aided simulation (Excel in our case). We adjust our contribution ratios $\overline{\beta}$ with an allowance γ and $\underline{\beta}$ with an allowance λ . The use of such allowance will enable us to test for the intervals where the contracts are attractive to the agents. The following tables below show the payoffs to the agents under different allowances levels for $\underline{\beta}$ and $\overline{\beta}$.

7. DISCUSSION

We discuss our results from the standpoint of the two contract types

7.1. High type contract

TABLE 3.

Payoff to the agents from high type contract when $\gamma = 10\%; \lambda = 10\%$

$\gamma = 10\%; \lambda = 10\%$	$\underline{\beta} \leq 78.9\%$	$\underline{\beta} \geq 78.9\%$
$\overline{\beta} < 20.54\%$	$634; -7; -267$	$634; -7; -267$
$20.54\% \leq \overline{\beta} \leq 66.9\%$	$323; 74; -142$	$323; 74; -142$
$\overline{\beta} > 66.9\%$	$-93; 136; -12$	$-93; 136; -12$

TABLE 4.

Payoff to the agents from high type contract $\gamma = 25\%; \lambda = 25\%$

$\gamma = 25\%; \lambda = 25\%$	$\underline{\beta} \leq 78.9$	$\underline{\beta} \geq 78.9$
$\overline{\beta} < 20.54\%$	$670; -18; -283$	$670; -18; -283$
$20.54\% \leq \overline{\beta} \leq 66.9\%$	$323; 74; -142$	$323; 74; -142$
$\overline{\beta} > 66.9\%$	$-258; 138; 20.54$	$-258; 138; 20.54$

TABLE 5.

Payoff to the agents from high type contract$\gamma = 10\%; \lambda = 25\%$

$\gamma = 10\%; \lambda = 25\%$	$\underline{\beta} \leq 78.9$	$\underline{\beta} \geq 78.9$
$\overline{\beta} < 20.54\%$	$634; -7; -267$	$634; -7; -267$
$20.54\% \leq \overline{\beta} \leq 66.9\%$	$323; 74; -142$	$323; 74; -142$
$\overline{\beta} > 66.9\%$	$-93; 136; -12$	$-93; 136; -12$

TABLE 6.

Payoff to the agents from high type contract $\gamma = 25\%; \lambda = 10\%$

$\gamma = 25\%; \lambda = 10\%$	$\underline{\beta} \leq 78.9$	$\underline{\beta} \geq 78.9$
$\overline{\beta} < 20.54\%$	$670; -18; -283$	$670; -18; -283$
$20.54\% \leq \overline{\beta} \leq 66.9\%$	$323; 74; -142$	$323; 74; -142$
$\overline{\beta} > 66.9\%$	$-258; 138; 20.54$	$-258; 138; 20.54$

TABLE 7.

Payoff to the agents from Low type contract $\gamma = 10\%; \lambda = 10\%$

$\gamma = 10\%; \lambda = 10\%$	$\underline{\beta} \leq 78.9$	$\underline{\beta} \geq 78.9$
$\overline{\beta} < 20.54\%$	$132; -7; -395$	$-18; -7; -269$
$20.54\% \leq \overline{\beta} \leq 66.9\%$	$51; 74; -314$	$-25; 74; -187$
$\overline{\beta} > 66.9\%$	$-85; 136; -252$	$-287; 136; -126$

The first thing to remark is that when the high type contract is selected, the outcome to the agent is the same regardless of the value of $\underline{\beta}$.

Second, the only interval where the high type contract is attractive to both the bank and the high type corporation is when $20.54\% \leq \overline{\beta} \leq 66.9\%$

TABLE 8.

Payoff to the agents from Low type contract $\gamma = 25\%; \lambda = 25\%$

$\gamma = 25\%; \lambda = 25\%$	$\underline{\beta} \leq 78.9$	$\underline{\beta} \geq 78.9$
$\overline{\beta} < 20.54\%$	$303; -18; -478$	$-479; -18; -103$
$20.54\% \leq \overline{\beta} \leq 66.9\%$	$210; 74; -386$	$-297; 74; -10$
$\overline{\beta} > 66.9\%$	$-128; 138; -322$	$-676; 138; 53$

TABLE 9.

Payoff to the agents from Low type contract $\gamma = 10\%; \lambda = 25\%$

$\gamma = 10\%; \lambda = 25\%$	$\underline{\beta} \leq 78.9$	$\underline{\beta} \geq 78.9$
$\overline{\beta} < 20.54\%$	$292; -7; -467$	$-490; -7; -92$
$20.54\% \leq \overline{\beta} \leq 66.9\%$	$210; 74; -386$	$-297; 74; -10$
$\overline{\beta} > 66.9\%$	$-126; 136; -324$	$-674; 136; 51$

TABLE 10.

Payoff to the agents from Low type contract $\gamma = 25\%; \lambda = 10\%$

$\gamma = 25\%; \lambda = 10\%$	$\underline{\beta} \leq 78.9$	$\underline{\beta} \geq 78.9$
$\overline{\beta} < 20.54\%$	$143; -18; -406$	$-7; -18; -280$
$20.54\% \leq \overline{\beta} \leq 66.9\%$	$51; 74; -314$	$-25; 74; -187$
$\overline{\beta} > 66.9\%$	$-87; 138; -250$	$-258; 138; 20.54$

Third, the only time the high type contract is attractive to the low type contract is when $\overline{\beta} > 66.9\%$. This choice however will never be set by the bank as it results in a negative payoff to the bank.

Fourth, when $\overline{\beta} < 20.54\%$ as λ increases from 10%to 25% the payoff to the bank from the high type contract increases from 634 to 670 while that of the corporation goes down fro -7 to -18. The reverse happens when $\overline{\beta} > 66.9\%$.

This means that the only feasible choice is that when $20.54\% \leq \overline{\beta} \leq 66.9\%$. This choice leads to the same payoff regardless of the level of the allowances λ and γ. This choice is also unattractive to the low type corporation.

7.2. Low type contract

Under all combination of λ and γ the only time the low type contract is attractive to the low type corporation is when $\overline{\beta} > 66.9\%$ and $\underline{\beta} > 78.9\%$. Yet this combination will never be chosen by the bank as it entails a negative payoff to the bank.

The only choice that is both attractive to the high type corporation and the bank is when $20.54\% \leq \overline{\beta} \leq 66.9\%$ and $\underline{\beta} < 78.9\%$. The payment from the low type corporation to the bank is improved by decreasing $\underline{\beta}$ by a bigger allowance λ. (when λ increased from 10% to 25% the payoff to the bank increased from 51 to 20.540 as in tables 7 and 8; 9 and 10).

From the discussion of high type contract and the low type contract, we can notice that the only strategy left for the bank then is to set:

$$20.54\% \leq \overline{\beta} \leq 66.9\% \text{ and } \underline{\beta} < 78.9\% \text{ and } \lambda = 25\%$$

This strategy is unattractive to the low type corporation but attractive to both the bank and the high type corporation. In fact the low type corporation will not be interested in any contracts as the choice of any one of them leads to it getting a negative payoff.

We are, then, having the following reduced form game from each type of contracts:

TABLE 11.
Payoff to the bank and to the low type corporation under high and low type contract and when $\lambda = 25\%$

		Low Type Corporation	
		P	NP
	$20.54\% \leq \overline{\beta} \leq 66.9\%$	323; −142	0; 0
Bank	$\underline{\beta} \leq 78.9\%$	210; −386	0; 0

TABLE 12.
Payoff to the bank and to the high type corporation under high and low type contract and when $\lambda = 25\%$

		High Type Corporation	
		P	NP
	$20.54\% \leq \overline{\beta} \leq 66.9\%$	323; 74	0; 0
Bank	$\underline{\beta} \leq 78.9\%$	210; 74	0; 0

7.3. Moral Hazards Premium (MHP)

It is clear that the lower type bank will not participate in any one of the contracts as it gets a negative payoff from both contracts. This way we have managed to reduce the problem of adverse selection as we are left with only the high type corporation. It is clear, from the tables above (11 and 12) that both contracts yield positive payoffs to the bank. Consequently, both the bank and the high type corporation will be willing to particiapte. Yet, the problem that remains is that the high type corporation can still choose a lower type contract as it yields the same payoff to it as the high type contract. This is a clear case of moral hazards that the after choosing the right corporation(Hight type), the latter might not perform as efficiently as required

To solve this problem, we propose the introduction of an incentive in the form of a premium. The agent can get such a premium only when proved to have performed as required. The maximum value of such incentive is the one that will equate the payoff to the bank under both contracts. We called this incentive: the "Moral Hazard Premium (MHP)". In our case, the bank will be willing to pay a maximum of $MPH = 323 - 210 = 113$ to encourage the high type corporation to choose a high type contract.

The final reduced game given the "Moral Hazard Premium (MHP)" looks like:

TABLE 13.

Payoff to the bank and to the high type corporation under high and low type contract and when $\lambda = 25\%$ and and MHP=113

		High Type Corporation	
		P	NP
	$20.54\% \leq \bar{\beta} \leq 66.9\%$	[210,323]; [74,187]	0; 0
Bank	$\underline{\beta} \leq 78.9\%$	210; 74	0; 0

We can notice that with the Moral Hazard premium, The high type contract is indeed attractive for both the bank and the high type corporation. Since, the payoff to the bank and to the high type corporation is given in an interval format, the moral hazard premium opens up a room for negotiation about how much premium is to be given.

8. CONCLUSION

In this paper, we use a game theory mechanism to corporate financing using profit and loss sharing (PLS) contracts. We employ mechanism design theory using two agents, a bank and a corporation which seeks financing through PLS mode. The corporation can be of two types (high or low). We employ the usage of two types of contracts (low and high). The purpose is to have each type of corporation choose a compatible type of contracts (i.e. high type corporation should choose high type contracts and vice versa).We found a theoretical evidence that this model helps well in minimizing asymmetric information in two ways. The first is the reduction of adverse selection. In our case the lower type of corporation is induced to exit the game as both contracts (low and high) were proved to be not attractive to it. Second, it helps in reducing moral hazards. In fact, the high type corporation might still choose the low type contracts unless if we introduce a moral hazard premium. Our results were also improved and clarified with the use of computer aided simulation. Our choice of output function and model parameters was generic in nature. We suggest that this model can be extended using specific sector data and then run the data in a multi-agent simulation interface such as Netlogo.

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