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Social Security, Intergenerational Transfers, and Growth

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The paper studies the effects of social security on the long-run per capita income growth and population growth. We incorporate the substitutional effect of social security on intergenerational transfers for old-age support within the family into an endogenous growth model. We find that under either an unfunded social security system or a fully funded social security system, the effects of social security on growth largely depend on parents' taste for the quantity of children. Social security may promote economic growth if increasing the social security tax reduces intergenerational transfers and population growth rate. Quantitative results verify the theoretical conclusions and show that an unfunded social security system is more effective in substituting intergenerational transfers within the family and hence is more likely to stimulate economic growth.

1. INTRODUCTION

Early studies on the growth effect of social security focus on one or both mechanisms of savings and bequests (Diamond, 1965; Feldstein, 1974; Barro, 1974). Later studies include education and human capital in their analysis, and some of them stress the mechanism of liquidity constraints (Kaganovich and Zilcha, 1999; Lambrecht, Michel, and Vidal, 2005; Glomm and Kaganovich, 2008). These mechanisms, however, may be reinforced or weakened by changes in population growth. For example, Barro (1974) notes that social security is neutral for economic growth because parents who care about children's welfare allocate resources between generations, giving rise to so-called Ricardian equivalence. Nonetheless, Becker and Barro (1988) argue that a key assumption of Barro's framework is that population grows at a constant rate. If changes in bequests affect the cost of raising children and hence the fertility rate, the effects of social security on growth through the mechanism of bequests would hardly stay neutral.

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From then on, there is a growing body of literature emphasizing the role of endogenous population growth. Most studies confirm Becker and Barro's conjecture (Lapan and Enders, 1990; Zhang, 1995). More recent works in this strand includes Zhang, Zhang, and Lee (2001), Van Groezen, Leers, and Meijdam (2003), Ehrlich and Kim (2007), and Yew and Zhang (2009). They emphasize alternative mechanisms like the mortality rate, the child allowance scheme, the fertility transition, the social welfare, etc.

However, these studies have largely ignored the motive of old-age support. Parents can be selfish rather than totally altruistic. They may raise and invest in children in the hope of getting material support when they are old. If they receive other benefits such as pensions, the old-age support motive may be dampened, which would further influence fertility choice and human capital investment in children. Indeed, few studies have identified this motivating force among parents which is referred to as "intergenerational transfers." Ehrlich and Lui (1991) introduce an endogenous population growth model with intergenerational transfers within the family and show that the old-age support motive plays a crucial role in the effects of mortality on the fertility rate and economic growth. Zhang and Zhang (1998) and Boldrin, De Nardi, and Jones (2005) are influential studies in the related literature, too. They identify the role of the old-age support motive, but fail to assess the possible changes in human capital investment, which is highly related to the fertility rate and largely determines per capita income growth. As first noted by Becker and Lewis (1973) and well documented by a series of later work (Becker, Murphy, and Tamura, 1990; Galor and Weil, 2000), parents face a trade-off between the quantity and quality of children, which implies that it may be misleading to focus solely on population growth or economic growth when studying the effects of social security. However, a comprehensive theoretical framework that incorporates intergenerational transfers within the family dealing with this issue is still missing in the literature.

In this paper, we combine the old-age support motive and the trade-off between the quantity and quality of children to investigate the mechanism of intergenerational transfers through which social security affects growth. We present an endogenous growth model that incorporates a government-run social security system and intergenerational transfers within the family. The model allows for the substitutional effect of social security on intergenerational transfers for old-age support within the family. We show that under either an unfunded social security system or a fully funded social security system, the effects of social security on growth largely depend on parents' taste for the quantity of children. Social security may promote economic growth if increasing the social security tax reduces intergenerational transfers and the population growth rate. However, if pensions are sufficient such that intergenerational transfers within the family are entirely replaced by the social security system, or if parents' taste for the quantity of children is sufficiently strong such that changes in intergenerational transfers within the family hardly affect the fertility rate, the fertility rate would not fall as the social security tax rate increases, and per capita income growth rate would decline.

To assess how quantitatively important social security is for growth, we conduct quantitative analysis on a grid of empirically plausible values for key parameters under two social security systems. The results verify the theoretical propositions and imply that an unfunded social security system is more effective in substituting intergenerational transfers within the family and hence is more likely to stimulate economic growth.

The remainder of the paper proceeds as follows. In Section 2, we present the model and analyze its balanced growth path under an unfunded social security system. Section 3 turns to the analysis under a fully funded social security system. Section 4 quantitatively assesses the growth effects of social security. Section 5 discusses alternative specifications of the model. Section 6 concludes the paper.

2. UNFUNDED SOCIAL SECURITY

2.1. The Model

We present an endogenous growth model in which the fertility rate, human capital formation, and intergenerational transfers within the family are jointly determined. The model economy is inhabited by overlapping generations of a large number of identical agents who live for three periods. The agent accumulates human capital when young, works, raises and invests in children in middle age, and lives in retirement in old age. The middle-aged agent raises and invests in children for old-age support and for the utility directly derived from children's companionship. Social security may affect the growth rates of per capita income and population by creating incentive effects on intergenerational transfers within the family and the agent's choices.

Let subscript t denote a period in time. The preference of a stand-in middle-aged agent is defined over the consumption in middle age c_t, the consumption in old age d_{t+1}, the number of children n_t, and the human capital of each child h_{t+1}:

$$\ln c_t + \ln n_t + \rho \ln h_{t+1} + \beta \ln d_{t+1}, \tag{1}$$

where $0 < \beta < 1$ is a discounting factor, and $\eta > 0$ and $\rho > 0$ measure the tastes for the quantity and quality of children respectively. Following Ehrlich and Lui (1991), we assume that the agent enjoys companionship or emotional benefits determined by the quantity and quality of children.

The middle-aged agent is endowed with one unit of time. To raise a child, the agent needs to spend $0 < v < 1$ units of time. She supplies the remaining $1-vn_t$ units of time to the labor market and earns $(1-vn_t)w_th_t$, where w_t is the effective wage rate and h_t is her human capital. She spends a τ fraction of income on the social security tax, a ϕ_{st} fraction on saving for old-age consumption and a χ_t fraction on the financial support for her parents. She also buys education e_tn_t for her children. Following De la Croix and Doepke (2003), we assume that price of education is the marginal productivity of the working generation, so the total spending on education is $w_t\bar{h}_te_tn_t$, where $\bar{h}_t$ is the average human capital of the working generation.

In old age at period $t+1$, the agent retires and receives returns from saving with rate R_{t+1}, pensions from social security system f_{t+1}, and transfers from children.

The budget constraints in two periods are written as

$$c_t + w_t\bar{h}_te_tn_t = w_th_t(1-vn_t)(1-\chi_t-\phi_{st}-\tau), \tag{2}$$

$$d_{t+1} = R_{t+1}w_th_t(1-vn_t)\phi_{st} + \chi_{t+1}w_{t+1}h_{t+1}(1-vn_{t+1})n_t + f_{t+1}. \tag{3}$$

The human capital of each child, h_{t+1}, depends on the education e_t, parents' human capital h_t, and average human capital of the working generation $\bar{h}_t$:

$$h_{t+1} = A(\bar{h}_te_t)^{\theta}h_t^{1-\theta}, \tag{4}$$

where $A > 0$ is a constant, and $0 < \theta < 1$ measures the contribution of education to human capital. We assume $\eta > \rho\theta$, which, as showed later, ensures a positive fertility rate in the case when intergenerational transfers are non-operative within the family.

The agent's utility maximization problem is to choose $c_t, n_t, e_t, \phi_{st}, d_{t+1}$, and h_{t+1} to maximize (1) subject to constraints (2)-(4). Performing the optimization yields the following first-order conditions for an interior solution:

$$\frac{1}{c_t} = \frac{\beta R_{t+1}}{d_{t+1}}, \tag{5}$$

$$\frac{vw_th_t(1-\chi_t-\tau) + w_t\bar{h}_te_t}{c_t} = \frac{\eta}{n_t} + \frac{\beta\chi_{t+1}w_{t+1}h_{t+1}(1-vn_{t+1})}{d_{t+1}}, \tag{6}$$

$$\frac{w_t\bar{h}_tn_t}{c_t} = \frac{\rho\theta}{e_t} + \frac{\theta\beta\chi_{t+1}w_{t+1}h_{t+1}(1-vn_{t+1})n_t}{d_{t+1}e_t}. \tag{7}$$

Following Becker and Murphy (1988), we treat intergenerational transfers within the family as an implicit contract between generations formed by social norms. The middle-aged agent transfers a fraction of income to her parents and expects that her children will do the same when she is old. A social welfare function is introduced to describe the contract:

$$\ln C_t^y + \lambda \ln C_t^o, \tag{8}$$

where $C_t^y = c_t N_t$ and $C_t^o = d_t N_{t-1}$ are the aggregate consumption of the middle-aged generation and the old-aged generation in period t respectively, and $\lambda > 0$ measures the relative weight of the welfare of the old to the young.

We assume the degree of intergenerational transfers within the family χ_t maximizes social welfare (8) under the non-negative constraint:

$$\chi_t \geq 0. \tag{9}$$

The first-order conditions are given by

$$d_t \geq \lambda n_{t-1} c_t, \tag{10}$$

$$(d_t - \lambda n_{t-1} c_t)\chi_t = 0. \tag{11}$$

The production factors include physical capital K_t and human capital H_t, which are employed by a representative firm in competitive market with the Cobb-Douglas technology:

$$Y_t = K_t^\alpha H_t^{1-\alpha}, \tag{12}$$

where Y_t is the output and $0 < \alpha < 1$ is a constant. Factor demands are determined by

$$\begin{aligned} R_t &= \alpha K_t^{\alpha-1} H_t^{1-\alpha}, && (13) \\ w_t &= (1-\alpha) K_t^\alpha H_t^{-\alpha}. && (14) \end{aligned}$$

The factor market clearing conditions are written as

$$\begin{aligned} K_t &= \varphi_{st-1} w_{t-1} h_{t-1} (1 - v n_{t-1}) N_{t-1}, && (15) \\ H_t &= h_t (1 - v n_t) N_t, && (16) \end{aligned}$$

where N_t is the population of the middle-aged agents, and we assume that physical capital is fully depreciated in one period.

Under an unfunded social security system, a government with a balanced budget taxes the middle-aged generation and transfers the tax to the old generation:

$$f_{t+1} = \tau w_{t+1} h_{t+1} (1 - v n_{t+1}) n_t. \tag{17}$$

2.2. Balanced Growth Path

We define

$$\phi_{ct} = \frac{c_t}{w_t h_t (1 - v n_t)}, \quad \phi_{et} = \frac{w_t \bar{h}_t e_t n_t}{w_t h_t (1 - v n_t)}, \quad \phi_{nt} = \frac{w_t h_t v n_t}{w_t h_t (1 - v n_t)},$$

where $\phi_{st}, \phi_{ct}, \phi_{et}$, and ϕ_{nt} denote a middle-aged agent's spending on saving, consumption, investment in children and raising children, in terms of labor income. On the balanced growth path, $\varphi_{st}, \varphi_{ct}, \varphi_{et}, \varphi_{nt}, w_t, R_t$, and χ_t are all constants. We remove subscript t to denote the level. It follows that per capita income, per capita capital, per capita consumption, and human capital grow at the same constant rate. We obtain two scenarios that describe the balanced growth path, depending on whether the intergenerational transfers within the family are entirely replaced by the social security system.[1]

Case 1: $\tau < \dfrac{\lambda}{1+\lambda+\rho\theta} - \dfrac{\alpha}{1-\alpha}\dfrac{1+\beta+\rho\theta}{1+\lambda+\rho\theta}$

$$\chi = \frac{\lambda - \frac{\alpha}{1-\alpha}(1+\beta+\rho\theta) - (1+\lambda+\rho\theta)\tau}{1+\lambda+\rho\theta+\beta\theta}, \tag{18}$$

$$\phi_s = \frac{a}{1-\alpha}\frac{\beta}{\lambda}, \tag{19}$$

$$\phi_c = \frac{1}{\lambda}\left(\frac{1}{1-\alpha}\frac{\lambda-\alpha\beta}{1+\lambda+\rho\theta} - \frac{\beta\theta}{1+\lambda+\rho\theta}\cdot\chi\right), \tag{20}$$

$$\phi_e = \frac{\theta}{\lambda}\left(\frac{\rho}{1-\alpha}\frac{\lambda-\alpha\beta}{1+\lambda+\rho\theta} + \frac{\beta(1+\lambda)}{1+\lambda+\rho\theta}\cdot\chi\right), \tag{21}$$

$$\phi_n = \frac{1}{\lambda}\frac{(\lambda-\alpha\beta)(\eta-\rho\theta) + [(1-\theta)(1+\lambda+\rho\theta) - \theta(\eta-\rho\theta)](1-\alpha)\beta\chi}{(1+\alpha\beta+\rho\theta)+\theta(1-\alpha)\beta\chi}. \tag{22}$$

Case 2: $\tau > \dfrac{\lambda}{1+\lambda+\rho\theta} - \dfrac{\alpha}{1-\alpha}\dfrac{1+\beta+\rho\theta}{1+\lambda+\rho\theta}$

$$\chi = 0, \tag{23}$$

$$\phi_s = \frac{(1-\tau)\frac{\alpha}{1-\alpha}\beta}{(1+\rho\theta)\tau + (1+\beta+\rho\theta)\frac{\alpha}{1-\alpha}}, \tag{24}$$

$$\phi_c = \frac{(1-\tau)(\tau+\frac{\alpha}{1-\alpha})}{(1+\rho\theta)\tau + (1+\beta+\rho\theta)\frac{\alpha}{1-\alpha}}, \tag{25}$$

$$\phi_e = \frac{\rho\theta(1-\tau)(\tau+\frac{\alpha}{1-\alpha})}{(1+\rho\theta)\tau + (1+\beta+\rho\theta)\frac{\alpha}{1-\alpha}}, \tag{26}$$

$$\phi_n = \frac{(\eta-\rho\theta)(\tau+\frac{\alpha}{1-\alpha})}{(1+\rho\theta)\tau + (1+\beta+\rho\theta)\frac{\alpha}{1-\alpha}}. \tag{27}$$

[1]We summarize all the qualitative results in the Appendix.

2.3. Comparative Statics

Performing comparative analysis on Equations (18)-(27) with respect to the social security tax rate τ, while holding other exogenous variables fixed, will provide insights into the effects of social security on growth, as well as the dependence of those on the key parameters of the model. We focus on changes in $\phi_n = vn/(1-vn)$ and $\phi_e/\phi_n = e$ because population growth rate changes in the same direction as ϕ_n, and per capita income growth rate changes in the same direction as ϕ_e/ϕ_n.

In Case 1, we obtain

$$\frac{d\chi}{d\tau} < 0, \quad \frac{d\phi_s}{d\tau} = 0, \quad \frac{d\phi_c}{d\tau} > 0, \quad \frac{d\phi_e}{d\tau} < 0,$$

$$\frac{d\phi_n}{d\tau} < 0 \iff \eta < \rho\theta + \frac{1-\theta}{\theta}(1+\alpha\beta+\rho\theta),$$

$$\frac{d\phi_e/\phi_n}{d\tau} > 0 \iff \eta < \rho\theta + \frac{1-\theta}{\theta}(1+\alpha\beta+\rho\theta)\Phi,$$

where $\Phi = \frac{(1+\lambda+\rho\theta)[(1+\alpha\beta+\rho\theta)(\lambda-\alpha\beta)\rho-(1+\lambda)(1-\alpha)^2\beta^2\theta\chi^2]}{(1+\lambda+\rho\theta)^2(1+\alpha\beta+\rho\theta)(\lambda-\alpha\beta)-\frac{1+\lambda}{\theta}[(1-\alpha)\beta\theta\chi-(\lambda-\alpha\beta)]^2}$.

We can prove that $0 < \Phi < 1$, so the condition ensuring $d(\phi_e/\phi_n)/d\tau > 0$ is stricter than $d\phi_n/d\tau < 0$, which is consistent with the result $d\phi_e/d\tau < 0$. We establish the following proposition.

Proposition 1. *In an economy with intergenerational transfers within the family, if parents raise and invest in children mainly for old-age support and if the taste for the quantity of children is sufficiently weak, then an increase in the social security tax rate under an unfunded social security system produces no change in the saving rate, increases the consumption rate of the middle-aged families, decreases intergenerational transfers within the family and promotes economic growth by decreasing population growth rate and increasing investment in each child.*

To obtain the intuition behind this proposition, we combine Equations (5)-(7) as:

$$\frac{vw_th_t(1-\chi_t-\tau)n_t + w_t\bar{h}_te_tn_t}{w_t\bar{h}_te_tn_t} = \frac{1}{\theta}\cdot\frac{\eta R_{t+1}c_t + \chi_{t+1}w_{t+1}h_{t+1}(1-vn_{t+1})w_t\bar{h}_te_tn_t}{\rho R_{t+1}c_t + \chi_{t+1}w_{t+1}h_{t+1}(1-vn_{t+1})w_t\bar{h}e_tn_t}. \tag{28}$$

The left-hand side of Equation (28) is the relative marginal cost of raising children to investing in children, and the right-hand side is the relative marginal return. The agent raises and invests in children for old-age support and for the utility directly derived from childrens companionship. An increase in the social security tax rate would partly replace intergenerational transfers within the family, causing negative incentive effect on raising and investing in children. Thus, the quantity and quality of children tend to decrease. However, due to the substitution relationship between the quantity and quality of children, a reduction in the quantity of children will result in a lower marginal cost of education and hence a positive effect on investment in human capital. A decrease in the quality of children will have a similar impact on the quantity of children. But if the taste for the quantity of children is weak, the old-age support motive for the quantity of children is relatively strong, making it more sensitive to the change in the social security tax rate. In Equation (28), the numerator of the right-hand side decreases by a larger amount than the denominator if the social security tax rate increases, implying a higher relative marginal return on the investment in children. Thus, the agent would substitute the quality for the quantity of children, which generates an increase in per capita income growth rate and a decrease in population growth rate. In contrast, if the taste for the quantity of children is strong, then population growth rate is relatively insensitive to changes in the social security tax rate. Then increasing social security tax rate would decrease the investment in each child and further yield a higher population growth rate and a lower per capita income growth rate.

If the social security tax rate is sufficiently high, the social security system may entirely replace intergenerational transfers within the family, as shown in Case 2. The comparative static results are obtained as:

$$\frac{d\phi_s}{d\tau} < 0, \quad \frac{d\phi_n}{d\tau} > 0, \quad \frac{d\phi_e/\phi_n}{d\tau} < 0;$$

if $\alpha(1+\rho\theta) > (1-2\alpha)\beta$, then $\frac{d\phi_c}{d\tau} < 0$, $\frac{d\phi_e}{d\tau} < 0$. Given that parameter α which measures the capital share of output, is rarely much lower than 1/3, we further assume that $\alpha(1+\rho\theta) > (1-2\alpha)\beta$ holds in the following analysis. In summary, the results are contained in Proposition 2:

PROPOSITION 2. *In an economy without intergenerational transfers within the family, an increase in the social security tax rate under an unfunded social security system decreases the consumption rate and the saving rate of middle-aged families and leads to slower economic growth by increasing the population growth rate and decreasing investment in children.*

In contrast to Proposition 1, Proposition 2 shows that if the social security system entirely replaces intergenerational transfers within the family, an increase in the social security tax rate would increase the fertility rate rather than decrease it. The institution is straightforward. The key mechanism through which social security affects growth is that the corresponding changes in intergenerational transfers within the family have impacts on the substitution relationship of the quantity and quality of children. In the absence of intergenerational transfers within the family, the social security tax is equivalent to the labor earning tax. An increased social security tax therefore results in a lower opportunity cost of raising children, a higher fertility rate, and in turn a lower investment in each child.

3. FULLY FUNDED SOCIAL SECURITY

3.1. The Model

In this section, we consider a model with a fully funded social security system in which a government saves the social security tax for each agent and returns it to her in old age. Now, budget constraint (3) becomes

$$d_{t+1} = R_{t+1} w_t h_t (1 - v n_t)(\phi_{st} + \tau) + \chi_{t+1} w_{t+1} h_{t+1} (1 - v n_{t+1}) n_t. \quad (29)$$

The capital market clearing condition (15) takes the form:

$$K_t = (\phi_{st} + \tau) w_{t-1} h_{t-1} (1 - v n_{t-1}) N_{t-1}. \quad (30)$$

Note that if the desired saving rate ϕ_{st} is positive, then the social security tax is neutral for growth because it only affects ϕ_{st}, and the total saving rate $\phi_{st} + \tau$ stays constant. Thus, we restrict our attention to a corner solution in which $\phi_{st} = 0$.

The agent's utility maximization problem is to choose c_t, n_t, e_t, d_{t+1}, and h_{t+1} to maximize (1) subject to Equations (2), (4), and (29). The first-order condition (7) still holds for e_t, but Equation (6) becomes

$$\frac{v w_t h_t (1 - \chi_t - \tau) + w_t \bar{h}_t e_t}{c_t} + \frac{\beta R_{t+1} v w_t h_t \tau}{d_{t+1}} = \frac{\eta}{n_t} + \frac{\beta \chi_{t+1} w_{t+1} h_{t+1} (1 - v n_{t+1})}{d_{t+1}}. \quad (31)$$

3.2. Balanced Growth Path

There are also two scenarios that describe the balanced growth path, depending on whether intergenerational transfers within the family are entirely replaced by the social security system.

Case 3: $\tau < 1 - \frac{\alpha}{1-\alpha}\frac{1+\rho\theta}{\lambda}$

$$\chi = \frac{\lambda - \frac{\alpha}{1-\alpha}(1+\rho\theta) - \lambda\tau}{1+\lambda+\rho\theta+\beta\theta}, \tag{32}$$

$$\phi_c = \frac{1}{\lambda}\left(\frac{\alpha}{1-\alpha} + \chi\right), \tag{33}$$

$$\phi_e = \frac{\theta}{\lambda}\left(\frac{\rho\alpha}{1-\alpha} + (\rho+\beta)\chi\right), \tag{34}$$

$$\phi_n = \frac{(\eta-\rho\theta)\alpha + [\beta(1-\theta)+(\eta-\rho\theta)](1-\alpha)\chi}{(1+\rho\theta+\beta)\alpha + (1+\rho\theta+\beta\theta)(1-\alpha)\chi}. \tag{35}$$

Case 4: $\tau > 1 - \frac{\alpha}{1-\alpha}\frac{1+\rho\theta}{\lambda}$

$$\chi = 0, \tag{36}$$

$$\phi_c = \frac{1}{1+\rho\theta}(1-\tau), \tag{37}$$

$$\phi_e = \frac{\rho\theta}{1+\rho\theta}(1-\tau), \tag{38}$$

$$\phi_n = \frac{\eta-\rho\theta}{1+\rho\theta+\beta}. \tag{39}$$

3.3. Comparative Statics

Performing comparative analysis on Equations (32)-(39) with respect to the social security tax τ, in Case 3 we obtain

$$\frac{d\chi}{d\tau} < 0, \quad \frac{d\phi_c}{d\tau} < 0, \quad \frac{d\phi_e}{d\tau} < 0, \quad \frac{d\phi_n}{d\tau} < 0,$$

$$\frac{d\phi_e/\phi_n}{d\tau} > 0 \iff \eta < \rho\theta + \Omega,$$

where $\Omega = \frac{(1-\theta)\beta[(1+\rho\theta+\beta)(\frac{\alpha}{1-\alpha})^2\rho - (1+\rho\theta+\beta\theta)(\rho+\beta)\chi^2]}{(1+\rho\theta+\beta\theta)(\rho+\beta)(\frac{\alpha}{1-\alpha}+\chi)^2 + (1-\theta)\beta^2(\frac{\alpha}{1-\alpha})^2}$.

Moreover, we assume $\chi < \sqrt{\frac{(1+\rho\theta+\beta)\rho}{(1+\rho\theta+\beta\theta)(\rho+\beta)}} \cdot \frac{\alpha}{1-\alpha}$ to ensure that $\Omega > 0$. A sufficient condition for this assumption is $\lambda < (1+\rho\theta+\sqrt{\frac{(1+\rho\theta+\beta)\rho}{(1+\rho\theta+\beta\theta)(\rho+\beta)}}) \cdot \frac{\alpha}{1-\alpha}$. We summarize the results as follows:

PROPOSITION 3. *In an economy with intergenerational transfers within the family, if parents raise and invest in children mainly for old-age support and the taste for the quantity of children is sufficiently weak, then an* *increase in the social security tax rate under a fully funded social security system decreases the consumption rate of middle-aged families, decreases intergenerational transfers within the family and promotes economic growth by decreasing the population growth rate and increasing investment in each child.*

The intuition is similar to that under an unfunded social security system. If the taste for the quantity of children is sufficiently weak, then the increased social security tax rate weakens old-age support motives for the quantity of children more than the quality of children, which in turn decreases the fertility rate and increases education through the substitution relationship between the quantity and quality of children.

In Case 4, the comparative static results are given by

$$\frac{d\phi_c}{d\tau} < 0, \quad \frac{d\phi_e}{d\tau} < 0, \quad \frac{d\phi_n}{d\tau} = 0, \quad \frac{d\phi_e/\phi_n}{d\tau} < 0.$$

The following proposition conveniently summarizes these results.

PROPOSITION 4. *In an economy without intergenerational transfers within the family, an increase in the social security tax rate under a fully funded social security system decreases the consumption rate of middle-aged families, produces no change in the population growth rate, and leads to slower economic growth by decreasing investment in each child.*

In the absence of intergenerational transfers within the family, a rise in the social security tax will result in a lower opportunity cost of raising children. In view of Equation (29) which differs from Equation (3) in the unfunded social security regime, the benefits from a fully funded social security system depend on the tax. Thus, the agent would not increase fertility because it will decrease benefits in old age.

4. QUANTITATIVE ANALYSIS

The previous two sections demonstrate that social security may influence the growth rates of population and per capita income under various circumstances. In this section, we quantitatively assess the extent of these effects and compare them under two social security systems.

Our strategy is to conduct quantitative analysis of ϕ_n and ϕ_e/ϕ_n on a grid of empirically plausible values for key parameters under the two social security systems. We still restrict ourselves to changes in the levels of ϕ_n and ϕ_e/ϕ_n.

We assume that one period lasts for 25 years. Following the literature, we choose $\beta = 0.6$, which implies that the discounting factor for one year is 0.98. Because $1-\theta$ determines the intergenerational persistency of earnings, which is estimated to range from 0.3 to 0.5 in empirical studies, we choose $\theta = 0.6$. Parameter α represents the income share of capital, so we set $\alpha = 0.5$. After the values of α and β are set, the value of λ determines the saving rate according to Equation (19). Our target for the saving rate is 0.2, implying $\lambda = 3.02$. There are few empirical studies that directly estimate parents' tastes for the quantity and quality of children. We begin with $\rho = 0.5$ and $\eta = 0.4$. We call the model determined by parameters with those values the benchmark model.

Table 1 presents the values of χ, ϕ_n, and ϕ_e/ϕ_n in the two social security systems and with various values of preferences for the quantity of children. To make it comparable, the tax rate under a fully funded social security system is defined as the tax rate minus the desired saving rate. For example, a 4% tax rate under a fully funded social security system indicates $\tau = 0.24$ in the model. The results verify Propositions 1-4. A gradual increase in the social security tax rate from 0 to 0.3 results in a decrease in intergenerational transfers within the family. This effect is more significant under an unfunded social security system than that under a fully funded social security system. When the tax rate is 0.3, intergenerational transfers within the family are entirely replaced by an unfunded social security system, while it is not true under a fully funded social security system.

As intergenerational transfers within the family decline, population growth rate decreases under both social security systems even when the taste for the quantity of children is enhanced. As predicted, when the social security system could not entirely replace intergenerational transfers within the family and when the taste for the quantity of children is sufficiently weak, per capita income growth rate rises along with the decline in intergenerational transfers within the family and the decline in population growth rate. However, this pattern may not hold if either of two conditions changes. As shown in Table 1, when intergenerational transfers within the family are entirely replaced under an unfunded social security system, or when the taste for the quantity of children is relatively stronger under a fully funded social security system, an increase in the social security tax may cause a decline in per capita income growth rate. Moreover, because intergenerational transfers within the family decrease more under an unfunded social security system than under a fully funded social security system, population growth rate falls more significantly as the social security tax increases, so per capita income growth rate is more likely to increase.

To test the sensitivity of population growth rate and per capita income growth rate to key parameters, we gradually change the value of one pa-

TABLE 1.

Intergenerational Transfers and Growth

Unfunded		$\eta = 0.35$		$\eta = 0.40$		$\eta = 0.45$	
τ	χ	φ_n	φ_e/φ_n	φ_n	φ_e/φ_n	φ_n	φ_e/φ_n
0.00	0.238	0.052	2.921	0.079	1.924	0.106	1.434
0.05	0.192	0.047	3.091	0.075	1.965	0.102	1.441
0.10	0.146	0.043	3.300	0.070	2.014	0.098	1.449
0.15	0.100	0.038	3.564	0.066	2.069	0.093	1.458
0.20	0.054	0.034	3.905	0.061	2.135	0.089	1.469
0.25	0.007	0.029	4.364	0.057	2.212	0.085	1.482
0.30	0.000	0.028	4.200	0.057	2.100	0.085	1.400
Fully Funded		$\eta = 0.35$		$\eta = 0.40$		$\eta = 0.45$	
τ	χ	φ_n	φ_e/φ_n	φ_n	φ_e/φ_n	φ_n	φ_e/φ_n
0.00	0.238	0.052	2.921	0.079	1.924	0.106	1.434
0.05	0.206	0.049	2.952	0.076	1.907	0.103	1.408
0.10	0.174	0.046	2.997	0.073	1.893	0.099	1.383
0.15	0.141	0.043	3.059	0.069	1.881	0.096	1.359
0.20	0.109	0.039	3.145	0.066	1.874	0.092	1.335
0.25	0.077	0.036	3.263	0.062	1.871	0.089	1.312
0.30	0.045	0.032	3.428	0.058	1.874	0.085	1.290

rameter while holding others fixed and then calculate the values of χ, ϕ_n, and ϕ_e/ϕ_n.[2]

We first change the value of θ from 0.5 to 0.7. The results show that a larger value of θ, or a lower degree of intergenerational mobility of income, makes parents less dependent on old-age support within the family, which further leads to a lower population growth rate and a higher per capita income growth rate. The differential patterns of intergenerational transfers within the family and population growth rate to an increase in social security tax between these values of θ are negligible. However, the per capita income growth rate increases by a larger amount when the value of θ is larger. Moreover, because intergenerational transfers within the family are still more sensitive to the tax rate under an unfunded social security system than they are under a fully funded social security system, the per capita income growth rate increases by a larger amount under an unfunded social security system.

Next, we change the value of ρ from 0.4 to 0.6. The result exhibits very similar patterns when the social security tax rate increases. A smaller value of ρ affects intergenerational transfers within the family and growth through the same mechanism as a larger value of θ, and hence, their re-

[2]We present the results from sensitivity check in the Appendix.

sponses to the social security tax rate do not differ significantly. When the value of α is changed from 0.3 to 0.5, the responses of intergenerational transfers within the family and growth to an increase in the social security tax would not significantly change.

Finally, we change the value of λ so that the saving rate is set from 0.15 to 0.25. A higher saving rate, or a smaller value of λ, makes it more likely for a social security system to entirely replace intergenerational transfers within the family. For example, when the saving rate is set to be 0.25, an unfunded social security system with a tax rate higher than 0.15 could reduce χ to zero. The response of population growth rate and per capita income to the increased social security tax rate when the saving rate is high follows a different pattern. Thus, population growth rate may increase under an unfunded social security system and stay constant under a fully funded system along with a rise in the social security tax rate, causing per capita income growth rate to decline.

5. DISCUSSION

In the previous sections, we examine various cases that may arise in the model. While this is a tractable way of introducing the mechanism of intergenerational transfers for social security analysis, there are several alternative specifications in the literature. We now discuss some examples in this section.

5.1. Lower Bound on the Fertility Rate

In some related studies, such as Ehrlich and Lui (1991), a lower bound on the fertility rate is often imposed to ensure that the model generates a positive population growth rate when the marginal cost of raising children is always greater than the marginal return. Although the previous sections focus only on interior solutions, we should not ignore that in some economies, the fertility rate cannot be lower than a certain level, or

$$n_t \geq \overline{n}. \tag{40}$$

We reconsider two social security systems with constraint (40). The first-order condition for n_t now becomes an inequality. More specifically, under an unfunded social security system, Equation (6) becomes

$$\frac{vw_t h_t(1-\chi_t-\tau)+w_t\bar{h}_t e_t}{c_t} \geq \frac{\eta}{n_t} + \frac{\beta\chi_{t+1}w_{t+1}h_{t+1}(1-vn_{t+1})}{d_{t+1}}, \tag{41}$$

Under a fully funded social security system, Equation (31) becomes

$$\frac{vw_t h_t(1-\chi_t-\tau)+w_t\bar{h}_t e_t}{c_t}+\frac{\beta R_{t+1} v w_t h_t \tau}{d_{t+1}} \geq \frac{\eta}{n_t}+\frac{\beta \chi_{t+1} w_{t+1} h_{t+1}(1-vn_{t+1})}{d_{t+1}}. \tag{42}$$

We still pay particular attention to the balanced growth path. Following a similar procedure, we find that the results of Cases 1-4 may still hold, if

$$\tilde{\phi}_n \geq \overline{\phi}_n = \frac{v\overline{n}}{1-v\overline{n}}, \tag{43}$$

where $\tilde{\phi}_n$ equals ϕ_n in Equation (22), (27), (35), and (39), corresponding to Cases 1-4. However, if condition (43) is violated, Equations (22), (27), (35), and (39) are all replaced by

$$\phi_n = \overline{\phi}_n. \tag{44}$$

Other results of Cases 1-4 still hold.[3]

When the fertility rate reaches its lower bound, it would no longer be affected by the social security tax. Per capita income growth rate could no longer increase with the social security tax rate, as shown in both tables. The previous sections outline the old-age support motive affected by social security in determining different growth rates of per capita income and population. In contrast, if the fertility rate is restricted to its lower bound, it is no longer the case that the quality of children increases because of the fall of the quantity, so the social security directly influences per capita income growth rate by decreasing investment in each child. In summary, a rise in the social security tax rate may promote economic growth only if parents are free to choose the quantity of children and receive transfers from children in old age.

5.2. Raising Cost Invariant to the Social Security Tax

In the preceding analysis, we adopt the specification that the cost of raising children is a fraction of parents' time, implying that the social security system taxing labor income directly influences the cost of raising children. However, other specifications where the cost of raising children is invariant to the social security tax can also be found in the literature.

When it takes a fraction v of labor income to raise a child, the agent's budget constraints under an unfunded social security system become:

$$c_t + w_t\bar{h}_t e_t n_t + v w_t h_t n_t = w_t h_t(1-\chi_t-\varphi_{st}-\tau), \tag{45}$$

$$d_{t+1} = R_{t+1} w_t h_t \varphi_{st} + \chi_{t+1} w_{t+1} h_{t+1} n_t + f_{t+1}, \tag{46}$$

[3] The results are summarized in the Appendix.

where φ_{st} denotes the saving rate. The government's balanced budget implies:

$$f_{t+1} = \tau w_{t+1} h_{t+1} n_t. \tag{47}$$

Under a fully funded social security system, Equation (45) still holds, while Equations (46) and (47) are replaced by:

$$d_{t+1} = R_{t+1} w_t h_t (\varphi_{st} + \tau) + \chi_{t+1} w_{t+1} h_{t+1} n_t. \tag{48}$$

To focus on the balanced growth path, it is convenient to define the middle-aged agent's spending on consumption, investment in and raising children in terms of labor income as follows:

$$\varphi_{ct} = \frac{c_t}{w_t h_t}, \quad \varphi_{nt} = v n_t, \quad \varphi_{et} = \frac{w_t \bar{h}_t e_t n_t}{w_t h_t}.$$

We solve for all the balanced growth path solutions with and without intergenerational transfers within the family under the two social security systems respectively. We then accordingly conduct comparative static analysis on φ_n and φ_e/φ_n.[4]

The results show that the effects of social security on intergenerational transfers within the family and on growth remain the same as summarized in Propositions 1 and 3 in an economy with intergenerational transfers within the family. Specifically, the condition that ensures the growth rate of per capita income increases along with social security tax rate becomes $\eta < \rho$. Although it is not the same as before, we can still conclude that if the taste for the quantity of children is sufficiently weak, an increase in the social security tax rate would speed up economic growth by decreasing population growth rate and increasing investment in each child. The only difference in the effects of social security on growth between the two specifications of raising cost is that when intergenerational transfers within the family are entirely replaced by the social security system, an increase in the social security tax rate is neutral, rather than negative, to per capita income growth rate.

Indeed, once we assume that the raising cost is invariant to the social security tax, we ignore the implicit mechanism in previous sections that the social security system directly affects the opportunity cost of raising children, which imposes a negative effect on population growth rate as the tax rate rises. Thus, in the present specification, an increase in the tax rate is more likely to cause population growth rate to decrease. However, because the key characteristic of the model giving rise to the positive effect of social security on economic growth is the substitutional effect on

[4]The results are presented in details in the Appendix.

intergenerational transfers within the family, which is still included in the present specification, the main findings in Propositions 1 and 3 still hold.

5.3. First-Best Allocation

There is a tradition in the literature of discussing the welfare effect of social security. In this subsection, we present the first-best allocation of the model and search for the optimal social security tax that generates the first-best growth.

We begin with a social planner economy in which there is neither a social security system operated by the government nor intergenerational transfers within the family. Instead, a social planner reallocates resources between generations to maximize the preference of each generation defined by Equation (1) and the social welfare function defined by Equation (8). Instead of the two-period budget constraints, the social planner now faces the resource constraint in each period:

$$(c_t + (1-\alpha)AK_t^{\alpha}H_t^{-\alpha}\bar{h}_t e_t n_t)N_t + d_t N_{t-1} + S_t = AK_t^{\alpha}H_t^{1-\alpha}, \tag{49}$$

where we retain the assumption that the unit cost of education equals the marginal productivity of the working generation. We use S_t to represent the resources allocated to investment, so the accumulation of physical capital accumulation function (Equation (15) or (30)) is replaced by

$$K_{t+1} = S_t. \tag{50}$$

We define

$$\gamma_{ct} = \frac{c_t N_t}{AK_t^{\alpha}H_t^{1-\alpha}}, \quad \gamma_{dt} = \frac{d_t N_{t-1}}{AK_t^{\alpha}H_t^{1-\alpha}}, \quad \gamma_{st} = \frac{s_t N_t}{AK_t^{\alpha}H_t^{1-\alpha}}.$$

The balanced growth path can now be written as

$$\gamma_c = \frac{1}{1+\lambda}\frac{1+\beta}{1+\beta+\alpha\beta+[\rho+(1-\alpha)\beta]\theta}, \tag{51}$$

$$\gamma_d = \frac{\lambda}{1+\lambda}\frac{1+\beta}{1+\beta+\alpha\beta+[\rho+(1-\alpha)\beta]\theta}, \tag{52}$$

$$\gamma_s = \frac{\alpha\beta}{1+\beta+\alpha\beta+[\rho+(1-\alpha)\beta]\theta}, \tag{53}$$

$$\phi_e = \frac{1}{1-\alpha}\frac{[\rho+(1-\alpha)\beta]\theta}{1+\beta+\alpha\beta+[\rho+(1-\alpha)\beta]\theta}, \tag{54}$$

$$\phi_n = \frac{\eta-\rho\theta+(1-\theta)(1-\alpha)\beta}{(1-\alpha)(1+\beta+\alpha\beta)+[\rho+(1-\alpha)\beta]\theta}. \tag{55}$$

In contrast to the results in the decentralized economy, it is apparent that qualitatively, the decentralized economy could achieve socially optimal levels of the saving rate, the fertility rate, and per capita income growth rate, given specified values of parameters and the social security tax. However, it is still unclear whether the decentralized economy could achieve the first-best allocation with plausible values of key parameters, so we resort to quantitative analysis.[5]

Note that parameter λ only affects consumption between generations in the first-best allocation. Hence our strategy is to start with the benchmark model except for $\lambda = 2.39$ to generate the same saving rate as in the socially optimal allocation and then compare the allocations with their first-best levels. The results imply that there is a large gap between population growth rate in a decentralized economy and its socially optimal level. Per capita income growth rate approaches to its socially optimal level when the tax rate is around 0.13, but never equals to its socially optimal level under an unfunded social security system. Although a lower value of parameter λ could generate a higher per capita income growth rate, it also leads to a lower population growth rate. This implies that an unfunded social security system could achieve the optimal per capita income growth rate level with a saving rate higher than its optimal level, but population growth rate goes further away from its optimal level. We also allow the values of other parameters to change over a wide range, but population growth rate in the decentralized economy is always lower than its optimal level.

In summary, a decentralized economy could generate the socially optimal per capita income growth rate with the specified social security tax and with a higher saving rate than its socially optimal level, but it cannot achieve the first-best allocations of saving, fertility and economic growth simultaneously. Intuitively, a person who lives for finite generations in the OLG economy cannot trade with others from all the generations and is hence constrained by the return on capital, which may force him to save more than the optimal level for old-age consumption, giving rise to the so-called dynamic efficiency problem. Diamond (1965) argues that the social security system is an effective tool to solve this problem because it acts as an intermediate agent assisting in the trade between different generations. However, this conclusion relies on the assumptions of constant population growth rate and savings alone determining per capita income, which do not hold in our model with endogenous population growth and endogenous economic growth. The magnitude of the effects of social security is restricted by the size of intergenerational transfers within the family. When the government adjusts the social security tax to achieve the optimal level

[5]We show the quantitative results in the Appendix.

of economic growth, it cannot simultaneously change the familys fertility rate to the optimal level.

6. CONCLUSION

This paper studies the effects of social security on the long-term growth rates of per capita income and population in an endogenous growth model in which intergenerational transfers within the family changes with respect to the social security tax. It shows that if parents raise and invest in children mainly for old-age support, and the taste for the quantity of children is sufficiently weak, then a rise in the social security tax rate would decreases intergenerational transfers within the family and promotes economic growth by decreasing population growth rate and increasing investment in each child. Moreover, in an economy without intergenerational transfers within the family, such effects vanish. Hence, the substitution effect of social security on intergenerational transfers within the family plays a crucial role here. The quantitative analyses show that an unfunded social security system is more likely to promote economic growth than a fully funded social security system because the former has a greater effect on decreasing intergenerational transfers within the family and population growth rate.

Our study implies that the relationship between intergenerational transfers operated by the government and intergenerational transfers within the family, should be emphasized in the social security system evaluation. In developing countries, the family's behavior is deeply influenced by traditional culture of old-age support, and elderly people depend heavily on the financial support provided by their children. Hence, old-age support is one of the most important motives for parents to raise and invest in children. In these countries, the substitution effect of social security on intergenerational transfers within the family is effective. A rise in the social security tax rate may not only lead to a decline in the fertility rate but also more rapid human capital accumulation and faster economic growth. Hence, more detailed empirical tests on the magnitude of intergenerational transfers within the family and the sensitivity to the social security system are needed.

This paper establishes an endogenous growth model with intergenerational transfers within the family. Several extensions are suggested. First, since the aging population has become a social issue, it is important to introduce the demographic transition into the model. Second, the retirement decision and public policy that are related to intergenerational transfers are other directions for future research.

APPENDIX

In this appendix, we present the key variables and comparative static results under different circumstances in Table A.1 and A.2. In Table A.3, we summarize the main results for the model with rearing cost invariant to the social security tax. The results of sensitivity check in the quantitative analysis are depicted in Figure A.1-A.4. Figure A.5 compares the decentralized economy with the first-best allocation.

TABLE 1.

Main Results for Unfunded Social Security System

<table>
<tr><td>Unfunded Regime</td><td colspan="2">$\tau < \frac{\lambda}{1+\lambda+\rho\theta} - \frac{\alpha}{1-\alpha}\frac{1+\beta+\rho\theta}{1+\lambda+\rho\theta}$</td><td colspan="2">$\tau > \frac{\lambda}{1+\lambda+\rho\theta} - \frac{\alpha}{1-\alpha}\frac{1+\beta+\rho\theta}{1+\lambda+\rho\theta}$</td></tr>
<tr><td>χ</td><td colspan="2">$\frac{\lambda - \frac{\alpha}{1-\alpha}(1+\beta+\rho\theta)-(1+\lambda+\rho\theta)\tau}{1+\lambda+\rho\theta+\beta\theta}$</td><td colspan="2">0</td></tr>
<tr><td>ϕ_s</td><td colspan="2">$\frac{\alpha}{1-\alpha}\frac{\beta}{\lambda}$</td><td colspan="2">$\frac{(1-\tau)\frac{\alpha}{1-\alpha}\beta}{(1+\rho\theta)\tau+(1+\beta+\rho\theta)\frac{\alpha}{1-\alpha}}$</td></tr>
<tr><td>ϕ_c</td><td colspan="2">$\frac{1}{\lambda}(\frac{1}{1-\alpha}\frac{\lambda-\alpha\beta}{1+\lambda+\rho\theta} - \frac{\beta\theta}{1+\lambda+\rho\theta}\cdot\chi)$</td><td colspan="2">$\frac{(1-\tau)(\tau+\frac{\alpha}{1-\alpha})}{(1+\rho\theta)\tau+(1+\beta+\rho\theta)\frac{\alpha}{1-\alpha}}$</td></tr>
<tr><td>ϕ_e</td><td colspan="2">$\frac{\theta}{\lambda}(\frac{\rho}{1-\alpha}\frac{\lambda-\alpha\beta}{1+\lambda+\rho\theta} + \frac{\beta(1+\lambda)}{1+\lambda+\rho\theta}\cdot\chi)$</td><td colspan="2">$\frac{\rho\theta(1-\tau)(\tau+\frac{\alpha}{1-\alpha})}{(1+\rho\theta)\tau+(1+\beta+\rho\theta)\frac{\alpha}{1-\alpha}}$</td></tr>
<tr><td rowspan="2">ϕ_n</td><td>$\tilde{\phi}_n$, if $\tilde{\phi}_n \geq \overline{\phi}_n$.</td><td>$\overline{\phi}_n$, if $\tilde{\phi}_n < \overline{\phi}_n$.</td><td>$\tilde{\phi}_n$, if $\tilde{\phi}_n \geq \overline{\phi}_n$.</td><td>$\overline{\phi}_n$, if $\tilde{\phi}_n < \overline{\phi}_n$.</td></tr>
<tr><td colspan="2">$\tilde{\phi}_n = \frac{1}{\lambda}\frac{(\lambda-\alpha\beta)(\eta-\rho\theta)+[(1-\theta)(1+\lambda+\rho\theta)-\theta(\eta-\rho\theta)](1-\alpha)\beta\chi}{(1+\alpha\beta+\rho\theta)+\theta(1-\alpha)\beta\chi}$</td><td colspan="2">$\tilde{\phi}_n = \frac{(\eta-\rho\theta)(\tau+\frac{\alpha}{1-\alpha})}{(1+\rho\theta)\tau+(1+\beta+\rho\theta)\frac{\alpha}{1-\alpha}}$</td></tr>
<tr><td rowspan="3">Comparative
Static Results</td><td colspan="2">$\frac{d\chi}{\partial\tau} < 0$, $\frac{d\phi_s}{\partial\tau} = 0$, $\frac{d\phi_c}{\partial\tau} > 0$, $\frac{d\phi_e}{\partial\tau} < 0$</td><td colspan="2">$\frac{d\phi_s}{d\tau} < 0$, $\frac{d\phi_c}{d\tau} < 0$, $\frac{d\phi_e}{d\tau} < 0$</td></tr>
<tr><td>$\frac{d\phi_n}{\partial\tau} < 0 \iff$
$\eta < \rho\theta + \frac{1-\theta}{\theta}(1+\alpha\beta+\rho\theta)$</td><td>$\frac{d\phi_n}{\partial\tau} = 0$</td><td>$\frac{d\phi_n}{d\tau} > 0$</td><td>$\frac{d\phi_n}{\partial\tau} = 0$</td></tr>
<tr><td>$\frac{d\phi_e/\phi_n}{\partial\tau} > 0 \iff$
$\eta < \rho\theta + \frac{1-\theta}{\theta}(1+\alpha\beta+\rho\theta)\Phi$</td><td>$\frac{d\phi_e/\phi_n}{d\tau} < 0$</td><td>$\frac{d\phi_e/\phi_n}{d\tau} < 0$</td><td>$\frac{d\phi_e/\phi_n}{d\tau} < 0 \iff$
$\alpha(1+\rho\theta) > (1-2\alpha)\beta$</td></tr>
</table>

TABLE 2.

Main Results for Full-funded Social Security System

<table>
<tr><td>Fully Funded
Regime</td><td colspan="2">$\tau < 1 - \frac{\alpha}{1-\alpha}\frac{1+\rho\theta}{\lambda}$</td><td colspan="2">$\tau < 1 - \frac{\alpha}{1-\alpha}\frac{1+\rho\theta}{\lambda}$</td></tr>
<tr><td>χ</td><td colspan="2">$\frac{\lambda-\frac{\alpha}{1-\alpha}(1+\rho\theta)-\lambda\tau}{1+\lambda+\rho\theta+\beta\theta}$</td><td colspan="2">0</td></tr>
<tr><td>ϕ_s</td><td colspan="2">0</td><td colspan="2">0</td></tr>
<tr><td>ϕ_c</td><td colspan="2">$\frac{1}{\lambda}(\frac{\alpha}{1-\alpha}+\chi)$</td><td colspan="2">$\frac{1}{1+\rho\theta}(1-\tau)$</td></tr>
<tr><td>ϕ_e</td><td colspan="2">$\frac{\theta}{\lambda}(\frac{\rho\alpha}{1-\alpha}+(\rho+\beta)\chi)$</td><td colspan="2">$\frac{\rho\theta}{1+\rho\theta}(1-\tau)$</td></tr>
<tr><td rowspan="2">ϕ_n</td><td>$\tilde{\phi}_n$, if $\tilde{\phi}_n \geq \overline{\phi}_n$.</td><td>$\overline{\phi}_n$, if $\tilde{\phi}_n < \overline{\phi}_n$.</td><td>$\tilde{\phi}_n$, if $\tilde{\phi}_n \geq \overline{\phi}_n$.</td><td>$\overline{\phi}_n$, if $\tilde{\phi}_n < \overline{\phi}_n$.</td></tr>
<tr><td colspan="2">$\tilde{\phi}_n = \frac{(\eta-\rho\theta)\alpha+[\beta(1-\theta)+(\eta-\rho\theta)](1-\alpha)\chi}{(1+\rho\theta+\beta)\alpha+(1+\rho\theta+\beta\theta)(1-\alpha)\chi}$</td><td colspan="2">$\tilde{\phi}_n = \frac{\eta-\rho\theta}{1+\rho\theta+\beta}$</td></tr>
<tr><td rowspan="3">Comparative
Static Results</td><td colspan="2">$\frac{d\chi}{d\tau} < 0$, $\frac{d\phi_c}{d\tau} < 0$, $\frac{d\phi_e}{d\tau} < 0$</td><td colspan="2">$\frac{d\phi_c}{d\tau} < 0$, $\frac{d\phi_e}{d\tau} < 0$</td></tr>
<tr><td>$\frac{d\phi_n}{d\tau} < 0$</td><td>$\frac{d\phi_n}{d\tau} = 0$</td><td>$\frac{d\phi_n}{d\tau} = 0$</td><td>$\frac{d\phi_n}{d\tau} = 0$</td></tr>
<tr><td>$\frac{d\phi_e/\phi_n}{d\tau} > 0 \iff \eta < \rho\theta + \Omega$</td><td>$\frac{d\phi_e/\phi_n}{d\tau} < 0$</td><td>$\frac{d\phi_e/\phi_n}{d\tau} < 0$</td><td>$\frac{d\phi_e/\phi_n}{d\tau} < 0$</td></tr>
</table>

FIG. 1. The Sensitivity Check for Parameter θ

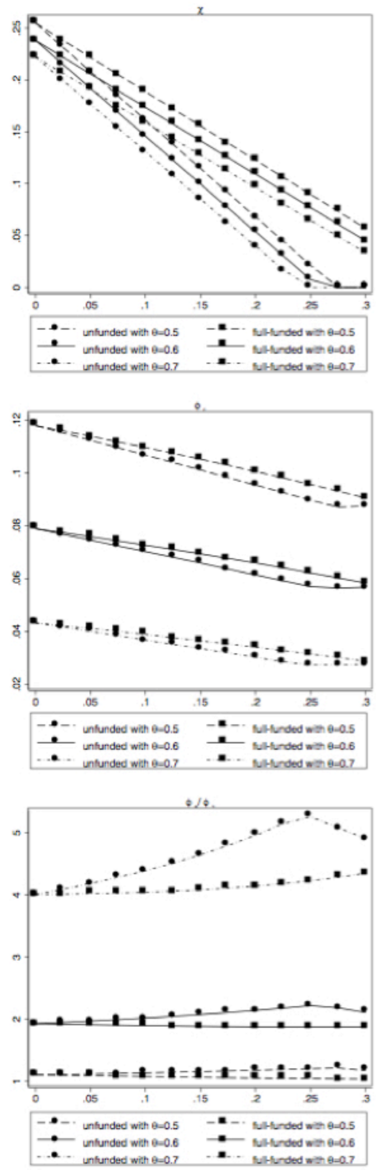

FIG. 2. The Sensitivity Check for Parameter α

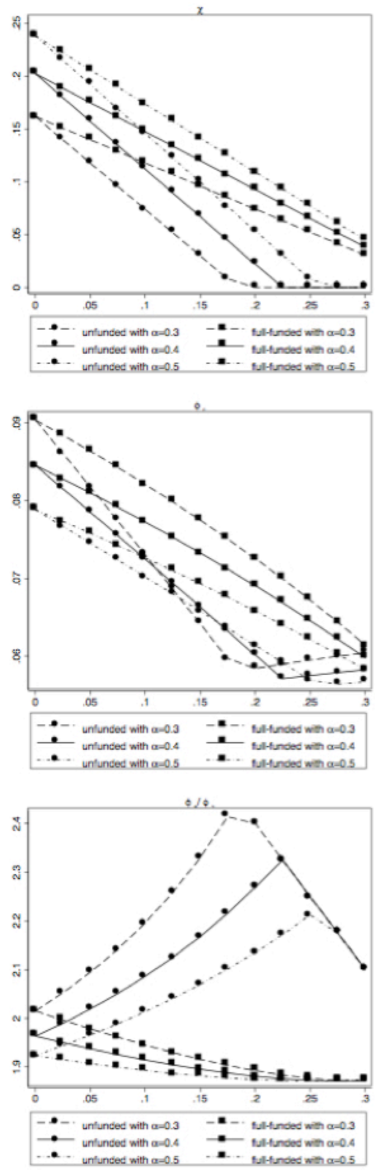

FIG. 3. The Sensitivity Check for Parameter ρ

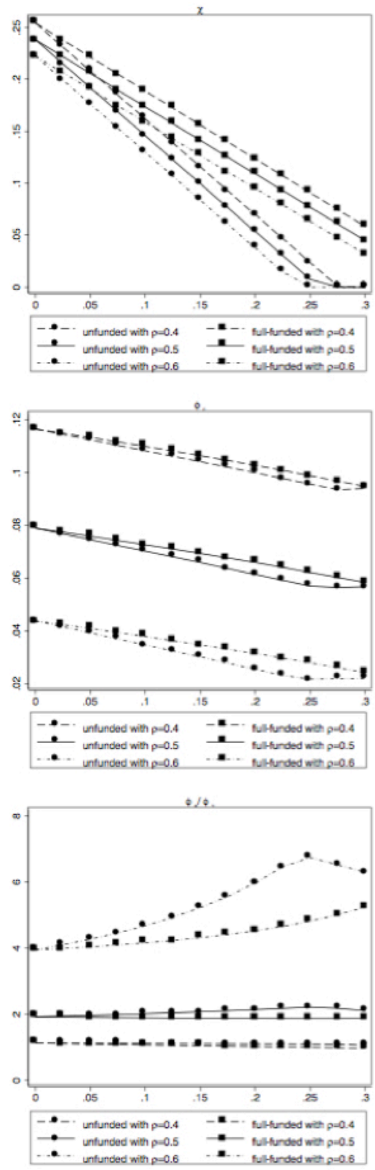

FIG. 4. The Sensitivity Check for Parameter λ

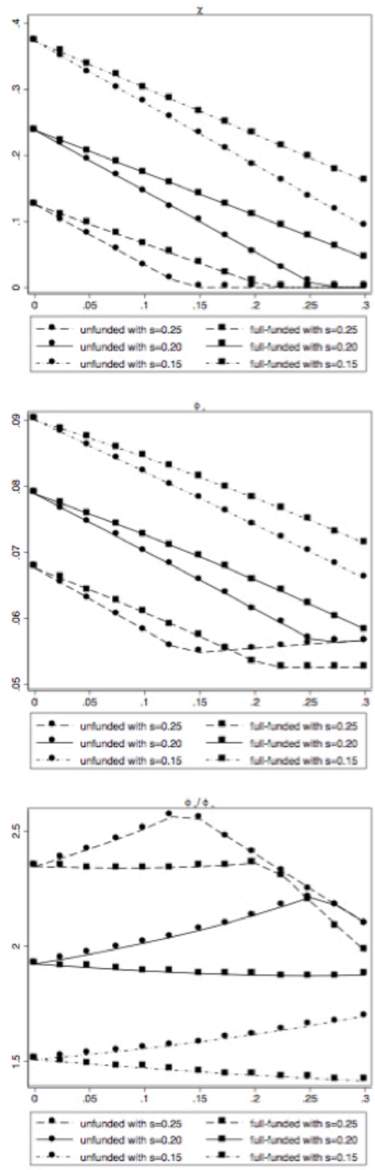

FIG. 5. Growth in the Decentralized Economy and the Socially Optimal Level

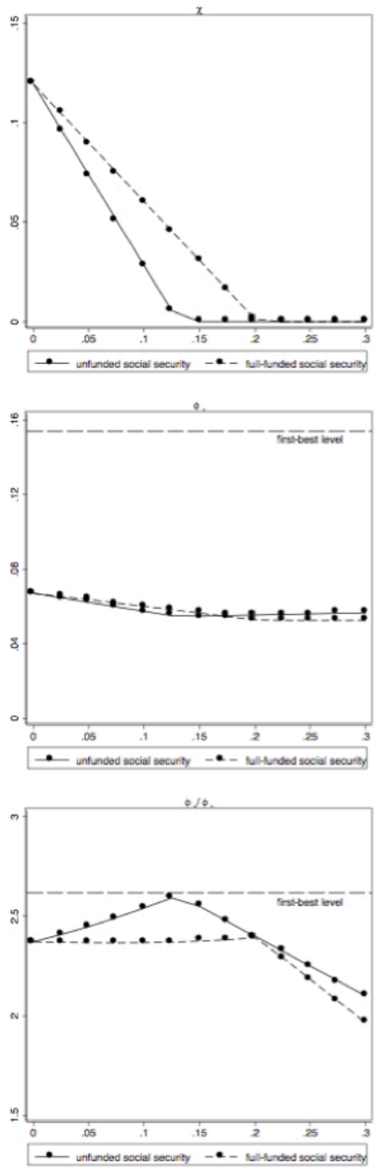

TABLE 3.

Main Results of Other Specifications of the Rearing Cost

	Unfunded Regime		Fully Funded Regime	
	$\tau < \frac{\lambda}{1+\lambda+\eta} - \frac{\alpha}{1-\alpha}\frac{1+\beta+\eta}{1+\lambda+\eta}$	$\tau > \frac{\lambda}{1+\lambda+\eta} - \frac{\alpha}{1-\alpha}\frac{1+\beta+\eta}{1+\lambda+\eta}$	$\tau < 1 - \frac{\alpha}{1-\alpha}\frac{1+\eta}{\lambda}$	$\tau > 1 - \frac{\alpha}{1-\alpha}\frac{1+\eta}{\lambda}$
χ	$\frac{\lambda - \frac{\alpha}{1-\alpha}(1+\beta+\eta)-(1+\lambda+\eta)\tau}{1+\lambda+\eta+\beta}$	0	$\frac{\lambda - \frac{\alpha}{1-\alpha}(1+\eta)-\lambda\tau}{1+\lambda+\eta+\beta}$	0
φ_s	$\frac{\alpha}{1-\alpha}\frac{\beta}{\lambda}$	$\frac{(1-\tau)\frac{\alpha}{1-\alpha}\beta}{(1+\eta)\tau+(1+\beta+\eta)\frac{\alpha}{1-\alpha}}$	0	0
φ_c	$\frac{1}{\lambda}(\frac{1}{1-\alpha}\frac{\lambda-\alpha\beta}{1+\lambda+\eta} - \frac{\beta}{1+\lambda+\eta}\chi)$	$\frac{(1-\tau)(\tau+\frac{\alpha}{1-\alpha})}{(1+\eta)\tau+(1+\beta+\eta)\frac{\alpha}{1-\alpha}}$	$\frac{1}{\lambda}(\frac{\alpha}{1-\alpha}+\chi)$	$\frac{1}{1+\eta}(1-\tau)$
φ_e	$\frac{\theta}{\lambda}(\frac{\rho}{1-\alpha}\frac{\lambda-\alpha\beta}{1+\lambda+\eta} + \frac{\beta(1+\lambda+\eta-\rho)}{1+\lambda+\eta}\chi)$	$\frac{\rho\theta(1-\tau)(\tau+\frac{\alpha}{1-\alpha})}{(1+\eta)\tau+(1+\beta+\eta)\frac{\alpha}{1-\alpha}}$	$\frac{\theta}{\lambda}(\frac{\rho\alpha}{1-\alpha}+(\rho+\beta)\chi)$	$\frac{\rho\theta}{1+\eta}(1-\tau)$
φ_n	$\frac{1}{\lambda}(\frac{\eta-\rho\theta}{1-\alpha}\frac{\lambda-\alpha\beta}{1+\lambda+\eta}$ $+\frac{\beta[(1-\theta)(1+\lambda)-(\eta-\rho)\theta]}{1+\lambda+\eta}\chi)$	$\frac{(\eta-\rho\theta)(1-\tau)(\tau+\frac{\alpha}{1-\alpha})}{(1+\eta)\tau+(1+\beta+\eta)\frac{\alpha}{1-\alpha}}$	$\frac{1}{\lambda}(\frac{\alpha}{1-\alpha}(\eta-\rho\theta)$ $+[\eta-\rho\theta+(1-\theta)\beta]\chi)$	$\frac{\eta-\rho\theta}{1+\eta}(1-\tau)$
Comparative Static Results	$\frac{d\chi}{d\tau}<0, \frac{d\varphi_s}{d\tau}=0, \frac{d\varphi_c}{d\tau}>0$	$\frac{d\varphi_s}{d\tau}<0, \frac{d\varphi_c}{d\tau}<0$	$\frac{d\chi}{d\tau}<0, \frac{d\varphi_c}{d\tau}<0$	$\frac{d\varphi_c}{d\tau}<0$
	$\frac{d\varphi_e}{d\tau}<0 \iff 1+\lambda+\eta-\rho>0$	$\frac{d\varphi_e}{d\tau}<0$	$\frac{d\varphi_e}{d\tau}<0$	$\frac{d\varphi_e}{d\tau}<0$
	$\frac{d\varphi_n}{d\tau}<0 \iff \eta<\rho+\frac{1-\theta}{\theta}(1+\lambda)$	$\frac{d\varphi_n}{d\tau}<0$	$\frac{d\varphi_n}{d\tau}<0$	$\frac{d\varphi_n}{d\tau}<0$
	$\frac{d\varphi_e/\varphi_n}{d\tau}>0 \iff \eta<\rho$	$\frac{d\varphi_e/\varphi_n}{d\tau}=0$	$\frac{d\varphi_e/\varphi_n}{d\tau}>0 \iff \eta<\rho$	$\frac{d\varphi_e/varphi_n}{d\tau}=0$

REFERENCES

Barro, Robert J., 1974. Are government bonds net wealth. *Journal of Political Economy* **82**, 1095-1117.

Becker, Gary S. and H. Gregg Lewis, 1973. On the interaction between the quantity and quality of children. *Journal of Political Economy* **81**, 279-288.

Becker, Gary S. and Robert J. Barro, 1988. A reformulation of the economic theory of fertility. *Quarterly Journal of Economics* **103**, 1-25.

Becker, Gary S. and Kevin M. Murphy, 1988. The family and the state. *Journal of Law of Economics* **31**, 1-18.

Becker, Gary S., Kevin M. Murphy, and Robert Tamura, 1990. Human capital, fertility, and economic growth. *Journal of Political Economy* **98**, 12-37.

Boldrin, Michele, Mariacristina De Nardi, and Larry E. Jones, 2005. Fertility and social security. NBER working paper number 11146.

de la Croix, David and Matthias Doepke, 2003. Inequality and growth: Why differential fertility matters. *American Economic Review* **93**, 1091-1113.

Diamond, Peter A., 1965. National debt in a neoclassical growth model. *American Economic Review* **55**, 1126-1150.

Ehrlich, Issac and Francis T. Lui, 1991. Intergenerational trade, longevity, and economic growth. *Journal of Political Economy* **99**, 1029-1059.

Ehrlich, Isaac and Jinyoung Kim, 2007. Social security and demographic trends: Theory and evidence from the international experience. *Review of Economic Dynamics* **10**, 55-77.

Feldstein, Martin, 1974. Social security, induced retirement, and aggregate capital accumulation. *Journal of Political Economy* **82**, 905-926.

Galor, Oded and David N. Weil, 2000. Population, technology, and growth: From Malthusian stagnation to the demographic transition and beyond. *American Economic Review* **90**, 806-828.

Glomm, Gerhard and Michael Kaganovich, 2008. Social security, public education and the growth-inequality relationship. *European Economic Review* **52**, 1009-1034.

Groezen, Bas van, Theo Leers, and Lex Meijdam, 2003. Social security and endogenous fertility: Pensions and child allowances as Siamese twins. *Journal of Public Economics* **87**, 223-251.

Koganovich, Michael and Itzhak Zilcha, 1999. Education, social security, and growth. *Journal of Public Economics* **71**, 289-309.

Lambrecht, Stephane, Philippe Michel, and Jean-Pierre Vidal, 2005. Public pensions and growth. *European Economic Review* **49**, 1261-1281.

Lapan, H. E. and W. Enders, 1990. Endogenous fertility, Ricardian equivalence, and the debt management policy. *Journal of Public Economics* **41**, 227-248.

Yew, Siew Ling and Jie Zhang, 2009. Optimal social security in a dynastic model with human capital externalities, fertility and endogenous growth. *Journal of Public Economics* **93**, 605-619.

Zhang, Jie, 1995. Social security and endogenous growth. *Journal of Public Economics* **58**, 185-213.

Zhang, Junsen and Junxi Zhang, 1998. Social security, intergenerational transfers, and endogenous growth. *Canadian Journal of Economics* **31**, 1225-1241.

Zhang, Junsen, Jie Zhang, and Ronald Lee, 2001. Mortality decline and long-run economic growth. *Journal of Public Economics* **80**, 485-507.