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Income effects, stabilization policy, and indeterminacy in one-sector models*

Yan Zhang†

The interrelations between indeterminacy and progressive income tax rules are discussed in a one-sector real business cycle model with capacity utilization, productive increasing returns, and the Jaimovich-Rebelo preferences that exhibit varying degrees of income effect. When the values of income effect are large and the other parameter values are plausible, a moderately progressive income tax schedule can destabilize the economy by generating local indeterminacy. Moreover, numerical examples show that when an income tax schedule with a progressivity feature destabilizes the economy, the degree of income effect and the minimum level of increasing returns required for local indeterminacy are negatively related. These results are in contrast to those obtained in Guo and Lansing (1998), in which a progressive tax schedule can stabilize the one-sector economy with increasing returns.

Key Words: Progressive income tax schedules; Income effects; Local indeterminacy.
JEL Classification Numbers: E30, E32, E62.

1. INTRODUCTION

In real business cycle (RBC) models with productive externalities (or increasing returns), local indeterminacy arises because a continuum of equilibrium paths converges to a common steady state.[1] Benhabib and Farmer (1994) conduct pioneer work in the area of literature that emphasizes how productive externalities generate indeterminacy. They assert that in a one-sector RBC model, sufficiently large productive externalities are needed to

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† Zhongnan University of Economics and Law. Any correspondence must be addressed to: Yan Zhang, Associate Professor, Wenlan School of Business, Zhongnan University of Economics and Law, 182 Nanhu Avenue, Wuhan, 430073, People's Republic of China. E-mail: z0004984@zuel.edu.cn.

[1] See Benhabib and Farmer (1999) for a survey.

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generate indeterminacy. The utility function featured in their work exhibits a positive income effect on the demand for leisure. However, Jaimovich (2008) argues that the indeterminacy result obtained by Benhabib and Farmer (1994) can be overturned if the utility function does not exhibit an income effect. In addition, a small amount of such effect can render indeterminacy possible. The preferences that exhibit the varying degrees of income effect in his study were first introduced by Jaimovich and Rebelo (2009; hereafter JR) and further modified by Nourry, Seegmuller, and Venditti (2013; hereafter NSV).

On the basis of Benhabib and Farmer (1994), Guo and Lansing (1998) discuss the interrelations between income tax schedules and indeterminacy. They suggest that indeterminacy is more likely to occur when tax policy becomes more regressive. By contrast, when tax policy becomes more progressive, determinacy is more likely to occur. In this work, we demonstrate that their results need to be reconsidered if both capacity utilization and the modified JR preferences proposed by NSV are taken into account.

Our contributions are twofold. First, we show that a moderately progressive income tax schedule can destabilize the economy when the large values of income effect are considered and the reasonably high level of increasing returns is used. On the basis of the levels of income tax progressivity computed by Dromel and Pintus (2008) for the U.S. economy in 1940-1993, we use numerical examples to demonstrate that an income tax schedule with moderate progressivity may destabilize the economy. This result holds when the magnitude of income effect is large and the other parameter values are plausible. Second, we show that the degree of income effect and the minimum level of externalities required for indeterminacy are negatively related when the model is indeterminate and the level of tax progressivity is (realistically) low. Our main results are in contrast to those obtained in Guo and Lansing (1998), in which they show that a progressive tax schedule can ensure stability in a one-sector economy with increasing returns or externalities.

In a one-sector growth model with endogenous consumption taxes and the JR preferences, NSV explain their indeterminacy results by using the intratemporal and intertemporal mechanisms between consumption and labor. In our case, the mechanism that delivers indeterminacy can be best understood through the following process. Suppose an agent expects the capital stock to be higher due to an increase in the marginal utility of wealth (or shadow price). Given the intratemporal mechanism between consumption and labor, optimistic expectation can boost the aggregate labor supply and output when aggregate increasing returns and strong income effects are present. The intratemporal substitution effect induces the agent to reduce his/her labor supply and consumption, while the intratemporal income effect increases the labor supply. When the intratemporal income effect dominates, the aggregate labor supply (and output) rises. At this moment, the (after-tax) marginal rate of return on capital is expected to increase, provided that the magnitudes of increasing returns are not too small and the tax policy is not too progressive. This trend reduces the shadow price to maintain the overall returns on capital, thereby making local indeterminacy possible[2].

The intertemporal mechanism behind the indeterminacy results is deduced from the first-order conditions of the household with respect to consumption and labor. We show that the variations of the labor supply and consumption are closely linked with those of the shadow price. To be more concrete, the magnitude of the increase in labor supply in the intertemporal mechanism is determined by the term known as the first effect. Moreover, the magnitude of the decrease in consumption is dictated by the term known as the second effect. In the calibrated examples, the first effect is positive and decreases in the income effect. However, the second effect varies from negative to positive and increases in the income effect. The intertemporal mechanism is valid only if the first effect is small and positive while the second effect is large and positive. This finding confirms that indeterminacy requires the intratemporal mechanism to be consistent with the mechanism that nonmonotonically relies on the degrees of income effect.

Given the mild tax progressivity, the minimum level of productive externality that leads to indeterminacy increases as the income effect decreases. We explain this result as follows. The intratemporal mechanism indicates that increasing the income effect enlarges the aggregate labor supply when the intratemporal income effect dominates. Consequently, the after-tax marginal rate of the return on capital increases, thus increasing the likelihood of indeterminacy. In addition, intensifying the degree of externality also increases the marginal rate of the return on capital, thus inducing indeterminacy more easily. Roughly speaking, increasing both the degree of income effect and the degree of externality can simultaneously strengthen the possibility of generating indeterminacy. Therefore, the minimum level of externality that leads to indeterminacy increases as the income effect decreases. Moreover, the intertemporal mechanism shows that the second effect rises in the degree of income effect. However, the first effect decreases in the degree of income effect. Hence, a larger degree of income effect makes the intertemporal mechanism become more likely. In other words, the range of externality associated with indeterminacy widens when the degree of income effect intensifies. More precisely speaking, the minimum

[2]In fact, a moderately progressive tax policy that reduces the higher marginal returns to capital increases the difficulty of raising indeterminacy.

level of externality compatible with indeterminacy decreases as the income effect increases.

On the basis of the preceding analysis, we can regard this research as a natural extension of Guo and Lansing (1998) and as a related work in existing studies because varying degrees of income effect are introduced into a one-sector Ramsey model. Given the presence of a fixed (but not too large) income effect, the intertemporal mechanism that nonmonotonically relies on the magnitudes of income effect is largely ignored. Following NSV (2013), we consider this feature in the model of Guo and Lansing (1998). We demonstrate that the results obtained in Guo and Lansing (1998) should be reconsidered and revisited.

We organize our paper as follows. In Section 2, we describe the model and focus on the analysis of the local dynamics. Then we discuss some calibrated examples in Section 3. In Section 4, we provide the intuition. Afterward, we present the discrete-time version of this model in Section 5. Lastly, in Section 6, we conclude the paper.

2. MODEL

The production function, which is similar to that in Wen (1998), is based on Benhabib and Farmer (1994). The technology exhibits constant private returns to scale, that is,

$$Y_t = EB_t (e_t K_t)^a N_t^{1-a},\ a \in (0,1)\,, \tag{1}$$

where K_t, N_t, and $e_t \in [0,1]$ denote the capital stock, labor supply, and capacity utilization rate, respectively. The term EB_t represents productive externalities and takes the following form:

$$EB_t = \left(\overline{e_t K_t}^{a} \bar{N}_t^{1-a}\right)^{\theta},\ \theta \geq 0, \tag{2}$$

where $\bar{K}_t$, $\bar{N}_t$, and $\bar{e_t}$ denote the economy-wide average levels of capital, labor supply, and capacity utilization rate, respectively. The firm takes the externality term as given. Thus, perfect competition in the factor market implies the following:

$$R_t = a\frac{Y_t}{K_t}, \tag{3}$$

and

$$W_t = (1-a)\,\frac{Y_t}{N_t}, \tag{4}$$

where R_t and W_t denote the rental and wage rates, respectively.

In the symmetric equilibrium, the consistency condition requires that $\overline{K_t} = K_t$, $\overline{N_t} = N_t$, and $\bar{e_t} = e_t$ hold for all t. Therefore, the social production function is given by

$$Y_t = (e_t K_t)^{a(1+\theta)} N_t^{(1-a)(1+\theta)}. \tag{5}$$

The representative agent maximizes his/her lifetime utility, that is,

$$\max_{C_t, N_t} \int_0^\infty e^{-\rho t} u\left(C_t, N_t\right) dt, \tag{6}$$

which is subject to the following capital accumulation equation:

$$\dot{K}_t = (1 - \tau_t)\left(R_t K_t + W_t N_t\right) - C_t - \delta_t K_t,\ K_0 \text{ given}, \tag{7}$$

where C_t denotes consumption, τ_t denotes the (average) income tax rate, and the depreciation rate (δ_t) satisfies $\delta_t = \frac{1}{\eta_1} e_t^{\eta_1}$, which is similar to Wen (1998). We assume that $\eta_1 > 1$.

Similar to NSV (2013), we consider the modified JR preferences that exhibit varying degrees of income effect:

$$u\left(C_t, N_t\right) = \frac{\left(C_t - B\frac{N_t^{1+\chi}}{1+\chi} C_t^{\gamma}\right)^{1-\sigma} - 1}{1-\sigma}. \tag{8}$$

where σ, B, and $\chi \geq 0$ are model parameters, and $\gamma \in [0, 1]$ is used to represent the degree of income effect. When $\gamma = 1$, the utility function becomes the King–Plosser–Rebelo (KPR) class of preferences. When $\gamma = 0$, the Greenwood–Hercovitz–Huffman (GHH) preferences exhibit no income effect. The low values of γ represent weak income effects[3].

Similar to Guo and Lansing (1998), the government budget constraint in each period is given by

$$G_t = \tau_t Y_t, \tag{9}$$

where G_t denotes government spending. Tax rate τ_t is set to satisfy the following schedule:

$$\tau_t = 1 - \eta\left(\frac{\overline{Y}}{Y_t}\right)^{\phi},\ \eta \in (0, 1],\ \phi \in [0, 1), \tag{10}$$

[3]As NSV noted, the JR preferences and NSV-type preferences result in the same steady state, and their local dynamics are equivalent.

where $Y_t = R_t K_t + W_t N_t$ denotes the agent's tax base, and $\overline{Y}$ denotes the steady-state value of income per head. Marginal tax rate is expressed as follows:

$$\tau_{mt} = \frac{\partial\left(\tau_t Y_t\right)}{\partial Y_t} = 1 - \eta\left(1-\phi\right)\left(\frac{\overline{Y}}{Y_t}\right)^{\phi}. \tag{11}$$

Two parameters η and ϕ determine the level and slope of the tax rule, respectively. If $\phi > (<)0$, the marginal tax rate is larger (smaller) than the average tax rate. At this time, the tax rule is said to be progressive (regressive). When $\phi = 0$, the tax rate is constant. Thus, the tax rule is flat. The restrictions on η and ϕ are imposed to ensure that an interior steady state exists. In addition, $0 < \tau < 1$ and $0 < \tau_m < 1$ hold at the steady state[4]. As in Chen and Guo (2013), the budget constraint of the household (7) in this research must be jointly concave in the predetermined (K_t) and free variables (C_t and N_t)[5]. This condition implies that $\phi \geq 0$.

The first-order conditions associated with the agent's problem (with respect to C_t, N_t, e_t, and K_t) are as follows:

$$u_C\left(C_t, N_t\right) = \Lambda_t, \tag{12}$$

$$-u_N\left(C_t, N_t\right) = \left(1-\tau_{mt}\right)\Lambda_t W_t, \tag{13}$$

$$\left(1-\tau_{mt}\right)\frac{aY_t}{e_t} = e_t^{\eta_1 - 1} K_t, \tag{14}$$

$$\dot{\Lambda}_t = \Lambda_t[\rho + \delta_t - \left(1-\tau_{mt}\right)R_t], \tag{15}$$

and

$$\lim_{t\to\infty} e^{-\rho t}\Lambda_t K_t = 0, \tag{16}$$

where Λ_t is the marginal utility of income.

The resource constraint for the economy is given by

$$Y_t = G_t + C_t + (\dot{K_t} + \delta_t K_t). \tag{17}$$

Tedious algebra shows that the equilibrium conditions can be characterized as follows:

$$\frac{1 - \gamma B \frac{N_t^{1+\chi}}{1+\chi} C_t^{\gamma-1}}{\left(C_t - B\frac{N_t^{1+\chi}}{1+\chi}C_t^{\gamma}\right)^{\sigma}} = \Lambda_t, \tag{18}$$

[4]The equilibrium after-tax interest rate $(1-\tau_{mt})R_t$ must strictly decrease with respect to K_t. This situation yields a lower bound on ϕ, that is, $\frac{a(1+\theta)-1}{a(1+\theta)}$.

[5]The second-order conditions of the agent's problem are satisfied.

$$\frac{BN_t^{\chi}C_t^{\gamma}}{1-\gamma B\frac{N_t^{1+\chi}}{1+\chi}C_t^{\gamma-1}}=\eta\left(1-\phi\right)\left(\frac{\overline{Y}}{Y_t}\right)^{\phi}\frac{\left(1-a\right)Y_t}{N_t}, \tag{19}$$

$$Y_t=A^{\frac{a(1+\theta)}{\eta_1-a(1+\theta)(1-\phi)}}K_t^{a(1+\theta)t_k}N_t^{(1-a)(1+\theta)t_n}, \tag{20}$$

$$\dot{\Lambda}_t=\Lambda_t\left[\rho+\delta_t-\eta\left(1-\phi\right)\left(\frac{\overline{Y}}{Y_t}\right)^{\phi}\frac{aY_t}{K_t}\right], \tag{21}$$

and

$$\dot{K}_t=\eta\overline{Y}^{\phi}Y_t^{1-\phi}-C_t-\delta_tK_t, \tag{22}$$

where $A=a\eta\left(1-\phi\right)\overline{Y}^{\phi}$ is a constant,[6] and t_k and t_n are defined as

$$t_k=\frac{\eta_1-1}{\eta_1-a\left(1+\theta\right)\left(1-\phi\right)},\ t_n=\frac{\eta_1}{\eta_1-a\left(1+\theta\right)\left(1-\phi\right)}.$$

In Eq. (20), coefficients $\alpha\equiv a\left(1+\theta\right)t_k$ and $\beta\equiv\left(1-a\right)\left(1+\theta\right)t_n$ in the reduced-form production function are usually perceived as the effective returns on capital and labor inputs, respectively. We obtain $t_k<1$ and $t_n>1$ because $\eta_1>1$ and $a\left(1+\theta\right)<1$.

We define a competitive equilibrium as follows. A set of prices $\{R_t,W_t\}$, a set of quantities $\{C_t,N_t,K_t,e_t\}$, and a fiscal policy $\{\tau_t,G_t\}$ constitute a CE if $\{C_t,N_t,K_t,e_t\}$ solves the agent's utility maximization problem, $\{K_t,N_t\}$ solves the firm's profit maximization problem, and the government's budget constraint holds under the fiscal policy $\{\tau_t,G_t\}$.

The bar variables denote steady-state values. $\dot{K}_t=0$ and $\dot{\Lambda}_t=0$ imply the following:

$$\eta\overline{Y}=\overline{C}+\overline{\delta K},$$

and

$$\frac{\overline{Y}}{\overline{K}}=\frac{\rho+\overline{\delta}}{a\eta\left(1-\phi\right)}.$$

We can derive the following results from these equations:

$$\overline{K}=M_0\overline{N}^{\frac{(1-a)(1+\theta)t_n}{1-a(1+\theta)t_k}}, \tag{23}$$

[6] From Eq. (14), the optimal rate of capacity utilization can be derived as follows:

$$e_t=[AK_t^{a(1-\phi)(1+\theta)-1}N_t^{(1-a)(1-\phi)(1+\theta)}]^{\frac{1}{\eta_1-a(1+\theta)(1-\phi)}}.$$

and

$$\overline{C} = M_1 \overline{N}^{\frac{(1-a)(1+\theta)t_n}{1-a(1+\theta)t_k}}, \tag{24}$$

where $M_0 = \left[A^{\frac{a(1+\theta)}{\eta_1 - a(1+\theta)(1-\phi)}} \frac{a\eta(1-\phi)}{\rho+\overline{\delta}}\right]^{\frac{1}{1-a(1+\theta)t_k}}$ and $M_1 = M_0[\frac{\rho+\overline{\delta}}{a(1-\phi)} - \overline{\delta}]$. Moreover, $\overline{\delta} = \frac{\frac{Y}{K} a\eta(1-\phi)}{\eta_1}$ holds at the steady state. This situation implies $\eta_1 = \frac{\rho+\overline{\delta}}{\overline{\delta}}$.

Given the value of $\overline{N}$, the steady-state values of other variables can be determined. To simplify our analysis, we normalize the steady state by setting $\overline{N} = 1$. The normalized steady state (NSS) is given by $(\overline{C}, \overline{K}, \overline{N}) = (C(1), K(1), 1)$, where $C(1) = M_1$ and $K(1) = M_0$. Parameter B can be easily pinned down from Eq. (19),

$$B^* = \frac{M_2}{M_1^{\gamma-1}[1 + \frac{\gamma M_2}{(1+\chi)}]}, \tag{25}$$

where $M_2 = \eta(1-\phi)(1-a)\frac{A^{\frac{a(1+\theta)}{\eta_1 - a(1+\theta)(1-\phi)}} M_0^{a(1+\theta)t_k}}{M_1}$[7].

Given that the modified JR preferences may not be locally concave at the NSS, we need the following lemma to allow the locally optimal solution to exist.

Lemma 1. *The modified JR preferences are locally concave at the NSS if and only if*

$$\sigma \geq \max\{\sigma_1(\gamma), \sigma_2(\gamma)\}, \tag{26}$$

where $\sigma_1(\gamma) = \frac{\gamma(\gamma+\chi)Q(\gamma)}{\gamma(1+\chi)Q(\gamma) + \frac{\gamma Q(\gamma)+\chi(1+\chi)}{(1+\chi)-Q(\gamma)}[(1+\chi)-\gamma Q(\gamma)]}$, *and* $\sigma_2(\gamma) = \frac{\gamma(1-\gamma)Q(\gamma)[(1+\chi)-Q(\gamma)]}{[(1+\chi)-\gamma Q(\gamma)]^2}$ *with* $Q(\gamma) = \frac{M_2}{[1+\frac{\gamma M_2}{(1+\chi)}]}$.

Proof. See Appendix A.1. This lemma is identical to the lemma 2 obtained by NSV (2013). ▮

Under the assumption stated above, the local dynamics around the NSS can be discussed, and our indeterminacy results can be compared with those obtained by Guo and Lansing (1998).

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[7] At the NSS, the steady-state value of output is $Y(1) = A^{\frac{a(1+\theta)}{\eta_1 - a(1+\theta)(1-\phi)}} K(1)^{a(1+\theta)t_k}$ with $A = a\eta(1-\phi)Y(1)^{\phi}$. This condition implies that $A = \{a\eta(1-\phi)[\frac{\rho+\overline{\delta}}{a\eta(1-\phi)}]^{-\frac{\phi a(1+\theta)t_k}{1-a(1+\theta)t_k}}\}^{\frac{1}{k_1}}$ holds, where $k_1 = \frac{k_2 - \phi a(1+\theta)}{k_2}$ and $k_2 = [\eta_1 - a(1+\theta)(1-\phi)][1 - a(1+\theta)t_k]$.

After the log-linearization method is used, the local dynamics of Eq. (18) to Eq. (22) around the NSS can be written as follows:

$$\begin{bmatrix} \dot{\widehat{\Lambda_t}} \\ \dot{\widehat{K_t}} \end{bmatrix} = J \begin{bmatrix} \widehat{\Lambda_t} \\ \widehat{K_t} \end{bmatrix}, \tag{27}$$

where $\widehat{\Lambda_t}$ and $\widehat{K_t}$ denote the log deviations of Λ_t and K_t from their respective steady states. Additionally, four elements of the Jacobian matrix J can be found in Appendix A.2[8]. When local indeterminacy arises, the determinant and trace of the Jacobian matrix must be positive and negative, respectively. In the following proposition, we derive the necessary and sufficient conditions for local indeterminacy.

PROPOSITION 1. *Given that Lemma 1 holds, two critical values, $\underline{\gamma} \in (0,1)$ and $\overline{\gamma} \in (\underline{\gamma},1]$, exist. $\underline{\phi} \in [0,1)$, $\overline{\phi} \in (\underline{\phi},1)$, $\overline{\rho} \in (0,+\infty)$, and $\underline{\theta} \in (0,+\infty)$ exist for any $\gamma \in (\underline{\gamma},\overline{\gamma}]$ such that local indeterminacy arises if and only if $\rho \in (0,\overline{\rho})$, $\theta \in (\underline{\theta},+\infty)$, and $\phi \in (\underline{\phi},\overline{\phi})$.*

Proof. See Appendix A.3. ▮

This proposition shows that local indeterminacy is not possible when the GHH preferences ($\gamma = 0$) are used. Furthermore, local indeterminacy with KPR preferences ($\gamma = 1$) may not occur if the local concavity condition fails. To be precise, the tax progressivity (ϕ) affects the value of $\max\{\sigma_1(\gamma),\sigma_2(\gamma)\}$, and the latter is crucial to our indeterminacy result. In this case, indeterminacy may not occur because improper values of ϕ may make $\max\{\sigma_1(\gamma),\sigma_2(\gamma)\}$ too large, and the concavity condition cannot be satisfied.

3. NUMERICAL EXAMPLES

On the basis of quarterly data, we use $(a,\overline{\delta},\rho) = (0.3, 0.025, 0.01)$ in the calibrated example. This value implies $\eta_1 = 1.4$. Given that the steady-state government spending to output ratio of the U.S. economy is 0.2, we set $\eta = 0.8$. We use the other parameter values according to the following facts:

[8]In the matlab codes, we use the symbolic toolbox to calculate the Jacobian matrix for precise results. In addition, the linearization method is used.

i) The empirical literature shows that many estimates of the elasticity of intertemporal substitution (EIS) in consumption lie between 0 and 2. The recent interval of the EIS in consumption is from 2 to 3[9].

ii) Many articles assume that the Frisch elasticity of the labor supply is infinite. However, Rogerson and Wallenius (2009) find that its macro value is between 2.25 and 3. Furthermore, Chetty et al. (2012) recommend a value of 0.5 when the intensive margin of labor supply is considered. However, Chetty et al. (2012) recommend a value of 0.25 when the extensive margin of labor supply is considered. The sum of the intensive and extensive elasticities is referred to as the aggregate hours elasticity (0.75), which is often adopted in calibrating representative-agent RBC models. Therefore, we use the estimates of aggregate hours elasticities in the following numerical example. Notably, the estimates obtained by Kim and Shapiro (2003), Pistaferri (2003), and Domeij and Floden (2006) validate that the Frisch elasticities for males are between 0.7 and 1[10].

The EIS in consumption and the Frisch elasticity of labor at the NSS are defined as follows:

$$-\frac{1}{EIS} \equiv \frac{u_{CC}}{u_C}C(1) = \frac{\gamma(1-\gamma)Q(\gamma)}{(\chi+1)-\gamma Q(\gamma)} - \frac{(\chi+1)-\gamma Q(\gamma)}{(\chi+1)-Q(\gamma)}\sigma, \tag{28}$$

and

$$\frac{1}{Frisch} = (\frac{1}{\varepsilon_{NN}} - \frac{1}{\varepsilon_{CN}}), \tag{29}$$

where $\frac{1}{\varepsilon_{NN}} \equiv \frac{u_{NN}}{u_N}N$; $\frac{1}{\varepsilon_{CN}} \equiv \frac{Nu_{CN}}{u_C}$; and u_N, u_{NN}, u_C, and u_{CN} are defined in Appendix A.1 and are all evaluated at the NSS[11].

In the numerical example, we set $\phi = 0.042$, $\chi = 0.54$, and $\sigma = 0.375$. Dromel and Pintus (2008) report that the tax progressivity levels in the United States range from 4% to 11% in 1940-1993, with an average value of roughly 6%. Therefore, we choose $\phi = 0.042$. We select $\sigma = 0.375$ to ensure that the EIS in consumption lies in the interval of $(2.6908, 2.9693)$. The values for the EIS in consumption match the newest interval recommended by Vissing-Jorgensen and Attanasio (2003). We choose $\chi = 0.54$ to ensure that the Frisch elasticity lies in the interval of $(0.7271, 0.8035)$. The values for the Frisch elasticity match the values recommended by Chetty et al. (2012). We determine that with such a value of ϕ, a mildly progressive income tax schedule can induce indeterminacy when the values of income

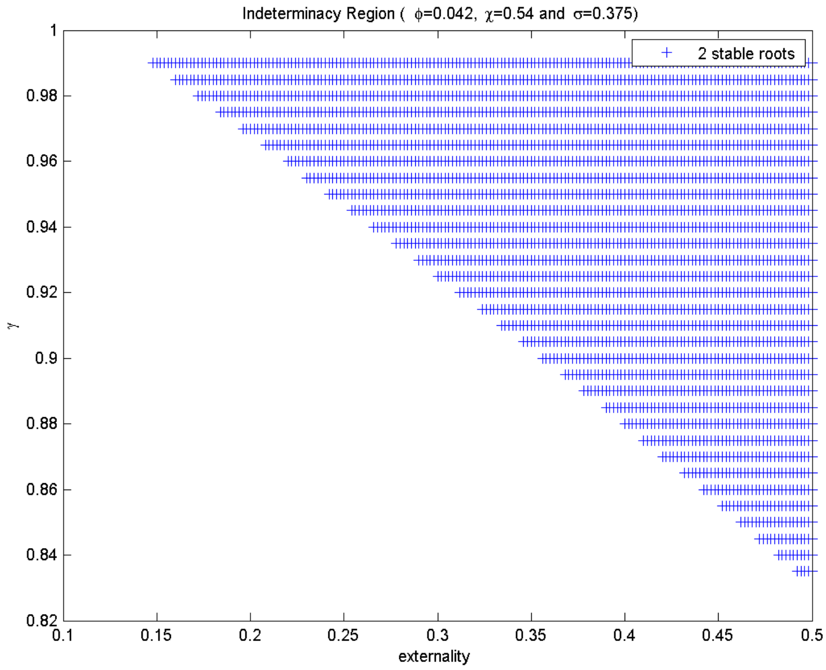
[9]See Vissing-Jorgensen and Attanasio (2003). In NSV (2013), this estimated interval is also used.

[10]Keane and Rogerson (2012) show that the Frisch elasticities that macroeconomists use are between 1 and 2.

[11]Holding tax rates and the marginal utility of income constant, we can derive the Frisch elasticity from the optimal conditions with respect to C and N.

effect (γ) lie in the interval of $(0.8350, 0.99)$[12]. We realize that in Abad et al. (2017, Figure 2), large values of γ [$\gamma \in (0.89, 0.97)$] are also required when balanced-budget rules with labor tax rates destabilize the economy. The minimal level of externality that induces indeterminacy is approximately 0.1480. Basu and Fernald (1997) and Burnside, Eichenbaum, and Rebelo (1995) estimate the aggregate returns to scale for the U.S. economy. Their estimates are within the range of 1.05 to 1.15. Therefore, indeterminacy might arise in our model, as the values of γ have not been clearly confirmed[13].

FIG. 1. Local dynamics. Blue area–indeterminacy.



4. DISCUSSION

We provide the intuition for our numerical results in this section. Similar to NSV, we first focus on the intratemporal choice between consumption

[12]Indeterminacy is not possible when $\gamma = 1$ because the local concavity condition is not satisfied.

[13]See Khan and Tsoukalas (2011) and Schmitt-Grohe and Uribe (2012).

and labor. The equilibrium labor supply curve is written as follows:

$$W_t = \eta\,(1-\phi)\,\overline{Y}^{\phi}\,\frac{BA^{\frac{a(1+\theta)\phi}{\eta_1 - a(1+\theta)(1-\phi)}} C_t^{\gamma} K_t^{a\phi(1+\theta)t_k} N_t^{\chi+(1-a)(1+\theta)\phi t_n}}{1-\gamma B \frac{N_t^{1+\chi}}{1+\chi} C_t^{\gamma-1}}. \quad (30)$$

Given that C_t and K_t are known, W_t is positively related to N_t in the (N_t, W_t) diagram, provided that $\phi \geq 0$. In other words, if the tax policy is progressive, an increase in C_t (or, K_t) causes the labor supply curve to shift to the left, thereby decreasing the equilibrium level of the labor supply.

Suppose the agent expects the capital stock to increase because of (an increase of) the marginal utility of wealth (or shadow price Λ_t). The agent reduces his/her labor supply because of the intratemporal substitution effect. This behavior shifts the labor supply curve to the left. Similarly, the agent needs to cut down consumption to increase capital stock. An intratemporal income effect comes into play and increases the equilibrium level of the labor supply. In other words, the intratemporal income effect shifts the supply curve to the right. If the income effect dominates the substitution effect, then the initial expectation generates a higher aggregate labor supply. Eq. (21) shows that the (after-tax) marginal rate of return on capital $[(1-\tau_{mt})\,R_t]$ is expected to increase if the magnitudes of increasing returns are not too small and the tax policy is not too progressive. Therefore, a price decline ($\dot{\Lambda}_t < 0$) should be required to maintain the overall returns on capital, which is equal to $\rho + \delta_t$. Thus, local indeterminacy arises as price declines and moves toward the steady state. As shown in Figure 1, indeterminacy requires large values of income effect if the magnitudes of increasing returns are small.

More precisely, Eq. (21) can be rewritten as follows:

$$\dot{\Lambda}_t = \Lambda_t \left[\rho - (1-\frac{1}{\eta_1}) \underbrace{\eta\,(1-\phi)\left(\frac{\overline{Y}}{Y_t}\right)^{\phi} \frac{aY_t}{K_t}}_{(1-\tau_{mt})R_t} \right].$$

We see that the after-tax rental rate is closely related to the term $K_t^{a(1-\phi)(1+\theta)t_k - 1} N_t^{(1-a)(1-\phi)(1+\theta)t_n}$. A mildly progressive tax policy in Example 1 can induce indeterminacy even if the magnitudes of increasing returns are small. In addition, capital stock is a predetermined variable. When optimistic expectations increase the aggregate labor supply, the after-tax rental rate can increase if $(1-a)(1-\phi)\,(1+\theta)\,t_n$ is relatively large. This relationship explains why an economy with a flat or moderately progressive tax schedule (small and positive ϕ) can be subject to local indeterminacy if the level of increasing returns (θ) is high and the income effect (γ) is sufficiently large[14]. This outcome occurs because when θ and γ are sufficiently large, the optimistic expectation can result in a substantial increase in the labor supply. Moreover, the after-tax rental rate can increase even if ϕ is small and positive. Similar to Wen (1998), the effective returns to scale can exceed the social returns to scale, thereby inducing indeterminacy for small magnitudes of externalities. With progressive taxes, a small and realistic degree of externalities, $\theta = 0.148$, is required for indeterminacy to occur. Without capacity utilization, indeterminacy needs extremely large increasing returns, as in Benhabib and Farmer (1994), even if the income effect is present[15].

Second, we focus on the intertemporal choice between consumption and labor. We differentiate Eq. (4) and obtain the following result[16]:

$$\frac{\dot{W}_t}{W_t} = [(1-a)(1+\theta)\,t_n - 1]\frac{\dot{N}_t}{N_t}.$$

On the basis of Eqs. (12) and (13), we can use the above equation to obtain the following intertemporal equation:

$$[1-(1-a)(1-\phi)(1+\theta)\,t_n + \frac{u_{NN}}{u_N}N]\frac{\dot{N}_t}{N_t} + \frac{C(1)u_{NC}}{u_N}\frac{\dot{C}_t}{C_t} = \frac{\dot{\Lambda}_t}{\Lambda_t}, \quad (31)$$

where $\frac{u_{NN}}{u_N}N \equiv \frac{1}{\varepsilon_{NN}}$; $\frac{C(1)u_{NC}}{u_N} \equiv -\frac{1}{\varepsilon_{NC}}$; and u_{NC}, u_N, and u_{NN} are defined in Appendix A.1 and evaluated at the NSS.

From the preceding analysis, we observe that the expected increase in the capital stock reduces the marginal utility of income Λ_t, i.e., $\frac{\dot{\Lambda}_t}{\Lambda_t} < 0$. At the same time, it decreases consumption ($\frac{\dot{C}_t}{C_t} < 0$) and increases labor supply ($\frac{\dot{N}_t}{N_t} > 0$). Indeterminacy occurs only if these variations can make Eq. (31) hold.

We define the first effect as $[1-(1-a)(1-\phi)(1+\theta)\,t_n - \frac{1}{\varepsilon_{NN}}]$, and the second effect as $-\frac{1}{\varepsilon_{NC}}$. In Example 1, we assume that $\theta = 0.2$. The first effect decreases in income effect parameter γ. However, the second effect increases in γ (see Figure 2). When $\gamma \in [0, \gamma_1)$, the second effect is negative, where γ_1 is close to 0.513. When $\gamma \in [\gamma_1, 0.99)$, its value is positive. A negative second effect ($\varepsilon_{NC} > 0$) is used to represent the Edgeworth complementarity between consumption and labor. We explain why indeterminacy is not possible when $\theta = 0.2$ and $\gamma \in [0, 0.967)$. Eq.

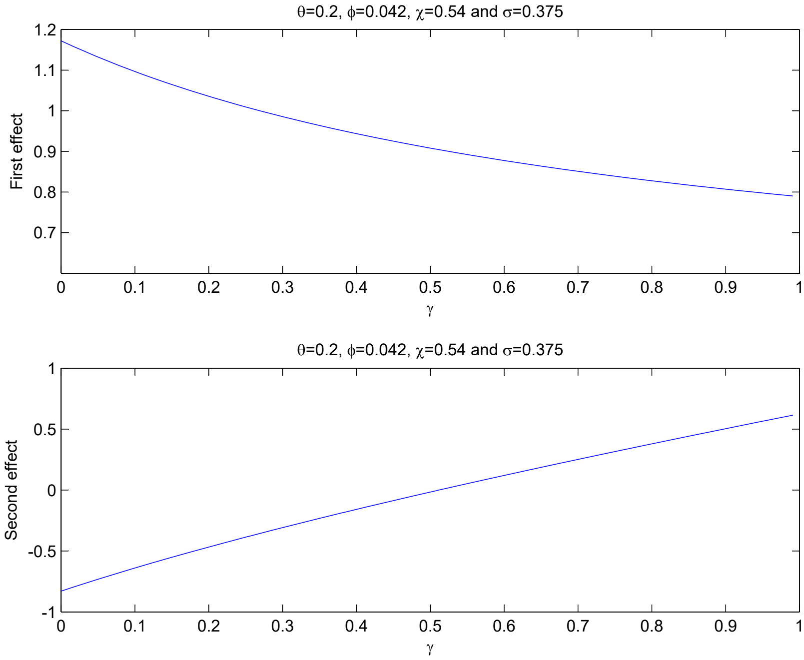
[14]The case of a flat tax schedule is discussed by Jaimovich (2008). In this case, we find that indeterminacy arises when $\theta = 0.098$.

[15]In Jaimovich (2008), a small amount of income effect can make indeterminacy possible because in his utility function, a state variable X_t is crucial to his result.

[16]The capital stock is a predetermined variable.

(31) cannot hold because of the very large and positive first effect and the negative or small positive second effect in this region. By contrast, when $\theta = 0.2$ and $\gamma \in [0, 968, 0.99)$, indeterminacy is possible because the first effect is small and positive and the second effect is large and positive. Thus, Eq. (31) can hold.

FIG. 2. The first and second effects ($\theta = 0.2$, $\chi = 0.54$, $\sigma = 0.375$, and $\phi = 0.042$)



Third, we explain why the minimum level of externality that induces indeterminacy increases as the income effect decreases given the mild tax progressivity. The intratemporal mechanism shows that increasing γ enlarges the aggregate labor supply when the income effect dominates. Thus, the after-tax rate of return on capital increases, thus making indeterminacy more likely. In addition, a high θ also increases the marginal rate of return on capital, thus inducing indeterminacy more easily. That is, increasing both γ and θ can simultaneously heighten the possibility of generating indeterminacy. Therefore, the minimum level of θ that induces indeterminacy increases as γ decreases. Next, the intertemporal mechanism shows that the second effect increases in γ, but the first effect decreases in γ. It implies that a larger value of γ increases the likelihood of the intertemporal mechanism. Thus, the range of θ associated with indeterminacy widens when γ becomes larger. In other words, the minimum level of θ compatible with indeterminacy decreases as γ rises.

Appendix A.3 shows that under the GHH preferences, indeterminacy cannot occur because neither the intratemporal mechanism nor the intertemporal mechanism mentioned above holds when $\gamma = 0$. Moreover, we provide the necessary and sufficient conditions for indeterminacy under the KPR preferences. We also find that indeterminacy cannot occur in the calibrated example when $\gamma = 1$ because the concavity conditions are not satisfied.

Lastly, we show that when ϕ increases, the minimal levels of γ and θ compatible with indeterminacy may rise. To be specific, $\phi = 0.06$, and the minimal levels of γ and θ associated with indeterminacy are 0.85 and 0.1680, respectively. These values of γ and θ clearly exceed their respective values in the above example. The reason for this result is as follows. First, when ϕ grows, larger values of γ and θ are required to increase the after-tax rate of return on capital, thus making the intratemporal mechanism hold. Second, we have difficulty determining how ϕ affects the first effect. Therefore, from the intertemporal mechanism, we cannot discern if the minimal levels of γ and θ compatible with indeterminacy rise when ϕ increases[17].

Two remaining issues could be tackled in the near future. First, the realistic values of γ have not been validated yet. Although the results obtained in Khan and Tsoukalas (2011) are in favor of a large γ, the results obtained in Schmitt-Grohe and Uribe (2012) show that the values of γ are close to zero. We suspect that the large values of γ required for local indeterminacy in Abad et al. (2017) also need to be validated. Therefore, the goal of this research is to conduct a theoretical study on local indeterminacy and to evaluate the degree of income effect, the level of tax progressivity, and the magnitude of productive externalities that might influence the occurrence of local indeterminacy. Second, the balanced-budget rule is crucial in the mechanism that delivers indeterminacy. In the seminal work of Schmitt-Grohe and Uribe (1997), a regressive labor income tax under the balanced-budget rule can lead to indeterminacy. However, under the same rule, a mildly progressive tax schedule coupled with mild increasing returns can induce indeterminacy as well. Therefore, we suspect that this rule alone can make indeterminacy more likely.

[17] Unlike Guo and Lansing (1998), we cannot analytically derive the minimal levels of θ and γ associated with indeterminacy as functions of ϕ. Therefore, numerical results show that when ϕ increases, the minimal levels of γ and θ compatible with indeterminacy may increase.

5. DISCRETE-TIME MODEL

In this section, we check whether our results still hold in the discrete-time formulation. NSV consider a discrete time version of their model. They cannot conclude that indeterminacy in the discrete-time model becomes more or less likely than in the continuous-time model. Unlike NSV, we show that indeterminacy becomes less likely in the discrete-time model because the minimal level of productive externality required for indeterminacy becomes larger for the same tax progressivity[18].

The problem of the agent in the discrete-time model is expressed as follows:

$$\max_{C_t, N_t, K_t} \sum_{t=0}^{+\infty} \beta_1^t u\left(C_t, N_t\right), \tag{32}$$

which is subject to

$$K_{t+1} = \left(1 - \tau_t\right)\left(R_t K_t + W_t N_t\right) - C_t + (1 - \delta_t) K_t, \ K_0 \text{ given}, \tag{33}$$

where $\beta_1 \in (0, 1)$ denotes the discount rate in the discrete-time model. The other notations are exactly the same as those in the continuous-time model.

We easily derive the following dynamic system:

$$\frac{1 - \gamma B \frac{N_t^{1+\chi}}{1+\chi} C_t^{\gamma-1}}{\left(C_t - B \frac{N_t^{1+\chi}}{1+\chi} C_t^{\gamma}\right)^{\sigma}} = \Lambda_{d,t}, \tag{34}$$

$$\frac{B N_t^{\chi} C_t^{\gamma}}{1 - \gamma B \frac{N_t^{1+\chi}}{1+\chi} C_t^{\gamma-1}} = \eta \left(1 - \phi\right) \left(\frac{\overline{Y}}{Y_t}\right)^{\phi} \frac{\left(1 - a\right) Y_t}{N_t}, \tag{35}$$

$$Y_t = A^{\frac{a(1+\theta)}{\eta_1 - a(1+\theta)(1-\phi)}} K_t^{a(1+\theta) t_k} N_t^{(1-a)(1+\theta) t_n}, \tag{36}$$

$$\Lambda_{d,t} = \beta_1 \Lambda_{d,t+1} \left[1 - \delta_{t+1} + \eta \left(1 - \phi\right) \left(\frac{\overline{Y}}{Y_{t+1}}\right)^{\phi} \frac{a Y_{t+1}}{K_{t+1}}\right], \tag{37}$$

and

$$K_{t+1} = \eta \overline{Y}^{\phi} Y_t^{1-\phi} - C_t + (1 - \delta_t) K_t, \tag{38}$$

where $\Lambda_{d,t}$ denotes the shadow price of the capital stock, and δ_t, A, t_k, and t_n have the same expressions as in Section 2.

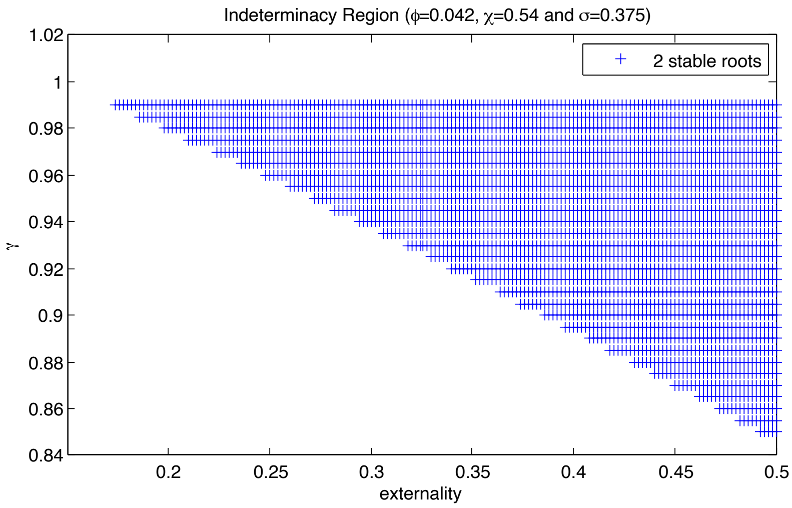
[18] At the same time, the region of γ associated with indeterminacy diminishes.

Using the same method as in Section 2, we derive the NSS in the above system. Let $\rho_1 = \frac{1}{\beta_1} - 1$, $M_{d0} = \left[A^{\frac{a(1+\theta)}{\eta_1 - a(1+\theta)(1-\phi)}} \frac{a\eta(1-\phi)}{\rho_1+\overline{\delta}}\right]^{\frac{1}{1-a(1+\theta)t_k}}$, and $M_{d1} = M_{d0}[\frac{\rho_1+\overline{\delta}}{a(1-\phi)} - \overline{\delta}]$. This NSS is given by $(\overline{C}_d, \overline{K}_d, \overline{N}_d) = (C_d(1), K_d(1), 1)$, where $K_d(1) = M_{d0}$ and $C_d(1) = M_{d1}$. Parameter B can be derived from Eq. (35),

$$B_d^* = \frac{M_{d2}}{M_{d1}^{\gamma-1}[1 + \frac{\gamma M_{d2}}{(1+\chi)}]}, \tag{39}$$

where $M_{d2} = \eta(1-\phi)(1-a)\frac{A^{\frac{a(1+\theta)}{\eta_1 - a(1+\theta)(1-\phi)}} M_{d0}^{a(1+\theta)t_k}}{M_{d1}}$[19].

FIG. 3. Local dynamics in the discrete-time model. Blue area–indeterminacy.



In the next step, we proceed as follows. For a given value of tax progressivity, we draw the regions of γ and θ in which indeterminacy exists. As in Section 3, we set $(a, \overline{\delta}, \beta_1) = (0.3, 0.025, 0.99)$ on the basis of quarterly data. We still assume that $\chi = 0.54$, $\sigma = 0.375$, and $\phi = 0.042$ hold. We find that the minimal level of productive externality required for generating indeterminacy becomes 0.174, which is larger than that in the continuous-time model. We also find that the region of γ associated with indeterminacy is smaller than that in the continuous-time model. Unlike NSV, we prove

[19] At the NSS, the value of output is $Y_d(1) = A^{\frac{a(1+\theta)}{\eta_1 - a(1+\theta)(1-\phi)}} K_d(1)^{a(1+\theta)t_k}$ with $A = a\eta(1-\phi)Y_d(1)^{\phi}$. Similarly, $A = \{a\eta(1-\phi)[\frac{\rho_1+\overline{\delta}}{a\eta(1-\phi)}]^{-\frac{\phi a(1+\theta)t_k}{1-a(1+\theta)t_k}}\}^{\frac{1}{k_1}}$ holds, where $k_1 = \frac{k_2 - \phi a(1+\theta)}{k_2}$ and $k_2 = [\eta_1 - a(1+\theta)(1-\phi)][1 - a(1+\theta)t_k]$.

that indeterminacy is less likely in our discrete-time model. The intuition for this result was provided by Anagnostopoulos and Giannitsarou (2013). Indeterminacy occurs only if the investment decisions made today are influenced by the expectations so powerfully that the initial expectations become self-fulfilling. A shorter period during which decisions are made strengthens the effect of the expectations on the choices today. Therefore, indeterminacy becomes more likely. In the continuous-time model, households make investment decisions instantly, thereby increasing the likelihood of indeterminacy.

6. CONCLUSION

We re-examine the indeterminacy results of Guo and Lansing (1998) in the standard one-sector RBC model with capacity utilization, aggregate productive externalities, and the JR preferences (2009) that exhibit a large range of income effect values. Under the preference specifications of additively-separable, Guo and Lansing (1998) argue that when the tax policy becomes more regressive, indeterminacy is more likely. By contrast, when the tax policy becomes more progressive, determinacy is more likely to occur. However, after we introduce the JR formulation and capacity utilization into their framework, we determine that the results obtained in Guo and Lansing (1998) must be reconsidered and revisited. We summarize our results as follows. First, when the degree of income effect is large and the level of productive externalities is reasonably high, a moderately progressive income tax schedule can destabilize the economy. Using the estimates of tax progressivity obtained by Dromel and Pintus (2008) for the U.S. economy, we demonstrate that an income tax schedule with moderate progressivity may destabilize the economy when the magnitude of income effect is large and the other parameter values are reasonable. Second, we show that when the model is indeterminate and the level of tax progressivity is realistically low, the degree of income effect and the minimum level of externalities required for indeterminacy are negatively related. Moreover, we consider the discrete-time version of this model and find that indeterminacy becomes less likely in this new environment.

APPENDIX A

A.1. Proof of lemma 1

Because $u(C_t, N_t)$ is not locally concave, we need find the local concavity conditions. Let $\Delta = C_t - B\frac{N_t^{1+\chi}}{1+\chi}C_t^{\gamma}$. The following restriction on (C_t, N_t) will be used. The time subscript will be deleted in order to save space.

Condition 1. In the feasible domain of (C, N), both $C > 0$ and $0 < \frac{N^{1+\chi}}{1+\chi} < \frac{C^{1-\gamma}}{B}$ hold.

Now, we examine the local concavity conditions of the utility function. $u(C, N)$ is locally concave if and only if $u_{CC} \leq 0$, $u_{NN} \leq 0$, and $u_{CC}u_{NN} - u_{CN}u_{NC} \geq 0$. First, the first and second derivatives of $u(C, N)$ are expressed as follows:

$$u_C(C, N) = \Delta^{-\sigma}\left(1 - \gamma B\frac{N^{1+\chi}}{1+\chi}C^{\gamma-1}\right) > 0, \tag{A1}$$

$$u_N(C, N) = -BN^{\chi}C^{\gamma}\Delta^{-\sigma} < 0, \tag{A2}$$

$$u_{CC} = \frac{u_C}{C}\left[\frac{\gamma(1-\gamma)C^{\gamma}B\frac{N^{1+\chi}}{1+\chi}}{C - \gamma C^{\gamma}B\frac{N^{1+\chi}}{1+\chi}} - \sigma\frac{C - \gamma C^{\gamma}B\frac{N^{1+\chi}}{1+\chi}}{C - B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}}\right], \tag{A3}$$

$$u_{NN}(C, N) = \frac{u_N}{N}\left(\sigma\frac{BN^{1+\chi}C^{\gamma}}{C - B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}} + \chi\right) \leq 0, \tag{A4}$$

$$u_{NC}(C, N) = \frac{u_N}{C}\left(\gamma - \sigma\frac{C - \gamma B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}}{C - B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}}\right), \tag{A5}$$

and

$$u_{CN}(C, N) = \frac{u_C}{N}\frac{BN^{1+\chi}C^{\gamma}}{C - \gamma B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}}\left(\sigma\frac{C - \gamma B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}}{C - B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}} - \gamma\right). \tag{A6}$$

Because $\gamma \in (0, 1)$ and $C > B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}$, we have $C > \gamma B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}$. The following conditions should be satisfied in order to make the local concavity conditions valid:

1. $u_{CC} \leq 0 \Leftrightarrow$

$$\frac{\gamma(1-\gamma)B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}}{C - \gamma B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}} \leq \sigma\frac{C - \gamma B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}}{C - B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}}. \tag{A7}$$

2. $u_{CC}u_{NN} - u_{CN}u_{NC} \geq 0$

$\Leftrightarrow$

$$\sigma\frac{\gamma B\frac{N^{1+\chi}}{1+\chi}C^{\gamma} + \chi C}{C - B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}} + \frac{\gamma\left[\sigma(1+\chi) - \gamma - \chi\right]B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}}{C - \gamma B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}} \geq 0. \tag{A8}$$

If condition 1 holds, $u_{NN} \leq 0$. Given the values of $\{\gamma, B\}$, the concavity conditions hold at the NSS, which implies that $\frac{\gamma(1-\gamma)B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}}{C-\gamma B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}} \leq \sigma\frac{C-\gamma B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}}{C-B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}}$ and $\sigma\frac{\gamma B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}+\chi C}{C-B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}} + \frac{\gamma[\sigma(1+\chi)-\gamma-\chi]B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}}{C-\gamma B\frac{N^{1+\chi}}{1+\chi}C^{\gamma}} \geq 0$.

From Eq. (25), we have

$$B^{*}C(1)^{\gamma-1} = \frac{M_2}{[1+\frac{\gamma M_2}{(1+\chi)}]} \equiv Q(\gamma). \tag{A9}$$

The local concavity condition becomes

$$\sigma \geq \sigma_1(\gamma) = \frac{\gamma(\gamma+\chi)Q(\gamma)}{\gamma(1+\chi)Q(\gamma)+\frac{\gamma Q(\gamma)+\chi(1+\chi)}{(1+\chi)-Q(\gamma)}[(1+\chi)-\gamma Q(\gamma)]},$$

and

$$\sigma \geq \sigma_2(\gamma) = \frac{\gamma(1-\gamma)Q(\gamma)[(1+\chi)-Q(\gamma)]}{[(1+\chi)-\gamma Q(\gamma)]^2}.$$

In other words, $\sigma \geq \max\{\sigma_1(\gamma), \sigma_2(\gamma)\}$.

In addition, the same four elasticities at the NSS are computed, as in NSV (2013).

$$\frac{1}{\varepsilon_{NN}} \equiv \frac{u_{NN}}{u_N}N = [\sigma\frac{(\chi+1)Q(\gamma)}{(\chi+1)-Q(\gamma)}+\chi],$$

$$\frac{1}{\varepsilon_{NC}} \equiv -\frac{C(1)u_{NC}}{u_N} = -[\gamma-\sigma\frac{(\chi+1)-\gamma Q(\gamma)}{(\chi+1)-Q(\gamma)}],$$

$$\frac{1}{EIS} \equiv -\frac{u_{CC}}{u_C}C(1) = -\frac{\gamma(1-\gamma)Q(\gamma)}{(\chi+1)-\gamma Q(\gamma)}+\frac{(\chi+1)-\gamma Q(\gamma)}{(\chi+1)-Q(\gamma)}\sigma,$$

$$\frac{1}{\varepsilon_{CN}} \equiv \frac{Nu_{CN}}{u_C} = \frac{(\chi+1)Q(\gamma)}{(\chi+1)-\gamma Q(\gamma)}[\sigma\frac{(\chi+1)-\gamma Q(\gamma)}{(\chi+1)-Q(\gamma)}-\gamma].$$

A.2. The elements in the Jacobian matrix

By log-linearizing (20), we have

$$\widehat{Y}_t = \alpha\widehat{K}_t + \beta\widehat{N}_t, \tag{A10}$$

where $\alpha = a\,(1+\theta)\,t_k$ and $\beta = (1-a)\,(1+\theta)\,t_n$.

By log-linearizing (18), we have

$$v_1\widehat{C}_t + v_2\widehat{N}_t = \widehat{\Lambda}_t, \tag{A11}$$

where $v_1 = \frac{U_{CC}}{U_C} C(1) = \frac{\frac{\gamma(1-\gamma)}{1+\chi} Q(\gamma)}{1-\frac{\gamma}{1+\chi} Q(\gamma)} - \sigma \frac{1-\frac{\gamma}{1+\chi} Q(\gamma)}{1-\frac{1}{1+\chi} Q(\gamma)}$ and $v_2 = \frac{U_{CN}}{U_C} \overline{N} = \frac{Q(\gamma)}{1-\frac{\gamma}{1+\chi} Q(\gamma)} [\sigma \frac{1-\frac{\gamma}{1+\chi} Q(\gamma)}{1-\frac{1}{1+\chi} Q(\gamma)} - \gamma]$.

By log-linearizing (19), we have

$$\widehat{N}_t [1 + \chi + \frac{\gamma Q(\gamma)}{1 - \gamma \frac{1}{1+\chi} Q(\gamma)}] + \widehat{C}_t [\gamma - \frac{\gamma (1-\gamma) \frac{1}{1+\chi} Q(\gamma)}{1 - \gamma \frac{1}{1+\chi} Q(\gamma)}] + (\phi - 1) \widehat{Y}_t = 0.$$

Using Eq. (A10), we have

$$v_3 \widehat{C}_t + v_4 \widehat{N}_t + \alpha (\phi - 1) \widehat{K}_t = 0, \tag{A12}$$

where $v_3 = \gamma - \frac{\gamma(1-\gamma) \frac{1}{1+\chi} Q(\gamma)}{1-\gamma \frac{1}{1+\chi} Q(\gamma)}$ and $v_4 = 1 + \chi + \frac{\gamma Q(\gamma)}{1-\gamma \frac{1}{1+\chi} Q(\gamma)} - (1 - \phi) \beta$.

Using Eqs. (A11) and (A12), we have

$$\widehat{N}_t = \frac{-v_3}{v_1 v_4 - v_2 v_3} \widehat{\Lambda}_t + \frac{v_1 \alpha (1 - \phi)}{v_1 v_4 - v_2 v_3} \widehat{K}_t = v_5 \widehat{\Lambda}_t + v_6 \widehat{K}_t, \tag{A13}$$

and

$$\widehat{C}_t = \frac{v_4}{v_1 v_4 - v_2 v_3} \widehat{\Lambda}_t - \frac{v_2 \alpha (1 - \phi)}{v_1 v_4 - v_2 v_3} \widehat{K}_t = v_7 \widehat{\Lambda}_t + v_8 \widehat{K}_t. \tag{A14}$$

Then, we have

$$\widehat{Y}_t = (\alpha + \beta v_6) \widehat{K}_t + \beta v_5 \widehat{\Lambda}_t.$$

Let $Y(1) = A^{\frac{a(1+\theta)}{\eta_1 - a(1+\theta)(1-\phi)}} K(1)^{\alpha}$. By log-linearizing Eq. (21), we obtain

$$\begin{aligned}
\dot{\widehat{\Lambda}}_t &= [-a\eta (1-\phi)^2 \frac{Y(1)}{K(1)} + \frac{a\eta (1-\phi)^2 Y(1)}{\eta_1 K(1)}] \widehat{Y}_t + [a\eta (1-\phi) \frac{Y(1)}{K(1)} - \frac{a\eta (1-\phi) Y(1)}{\eta_1 K(1)}] \widehat{K}_t \\
&= J_{11} \widehat{\Lambda}_t + J_{12} \widehat{K}_t,
\end{aligned}$$

where

$$J_{11} = -\rho (1-\phi) \beta v_5,$$

and

$$J_{12} = -\rho [(1-\phi)(\alpha + \beta v_6) - 1].$$

By log-linearizing Eq. (22), we obtain

$$\begin{aligned}
\dot{\widehat{K}}_t &= [\eta (1-\phi) \frac{Y(1)}{K(1)} - a\eta (1-\phi)^2 \frac{Y(1)}{\eta_1 K(1)}] \widehat{Y}_t - [\eta \frac{Y(1)}{K(1)} - \frac{C(1)}{K(1)} - a\eta (1-\phi) \frac{Y(1)}{\eta_1 K(1)}] \widehat{K}_t - \frac{C(1)}{K(1)} \widehat{C}_t, \\
&= J_{21} \widehat{\Lambda}_t + J_{22} \widehat{K}_t,
\end{aligned}$$

where

$$J_{21} = [\frac{\rho + \overline{\delta}}{a} - \overline{\delta}(1-\phi)](\beta v_5 - \frac{v_7}{1-\phi}),$$

and

$$J_{22} = [\frac{\rho + \overline{\delta}}{a} - \overline{\delta}(1-\phi)](\alpha + \beta v_6 - \frac{v_8}{1-\phi}).$$

A.3. Proof of Proposition 1

The trace of the Jacobian matrix can be written as

$$\mathcal{T} = \frac{1}{\Delta}\{-\rho(1-\phi)\beta\Delta_2 + \alpha[\frac{\rho+\overline{\delta}}{a} - \overline{\delta}(1-\phi)](\Delta_1 - \frac{1}{EIS} + \frac{1}{\varepsilon_{CN}})\}, \quad \text{(A15)}$$

where $\Delta_1 = \varepsilon_{CN}^{-1}\varepsilon_{NC}^{-1} - \varepsilon_{CC}^{-1}\varepsilon_{NN}^{-1}$, $\Delta_2 = \varepsilon_{NC}^{-1} - \varepsilon_{CC}^{-1}$, and $\Delta = -\varepsilon_{CC}^{-1}[1-(1-\phi)\beta] + \Delta_1$. Since the utility function is concave, $EIS \geq 0$ and $\Delta_1 \leq 0$ hold. From the normality condition, we see that $\Delta_2 \leq 0$ and $\varepsilon_{NN}^{-1} - \varepsilon_{CN}^{-1} \geq 0$ hold.

When the determinant is positive, we see that

$$\frac{1}{\Delta}\{(\frac{1}{1-\phi} - \alpha)[1-(1-\phi)\beta - \varepsilon_{CN}^{-1} + \varepsilon_{NN}^{-1}] + \beta\frac{1}{EIS} - \beta\frac{1}{\varepsilon_{NC}} - \alpha\beta(1-\phi)\} > 0. \quad \text{(A16)}$$

When the local concavity condition holds, we study the following two cases.

(i) When $\gamma = 0$, the trace in this GHH case becomes

$$\mathcal{T} = \frac{-\alpha\sigma}{\Delta}[\frac{\rho+\overline{\delta}}{a} - \overline{\delta}(1-\phi)](1+\chi).$$

The negative trace implies that $\phi \in (\underline{\phi}^0, \overline{\phi}^0)$ holds, where $\underline{\phi}^0 = 1 - \frac{(1+\chi)(\rho+\overline{\delta})}{(1-a)(\rho+\overline{\delta})+a\overline{\delta}(1+\chi)}$ and $\overline{\phi}^0 = 1 - \frac{(1+\chi)(\rho+\overline{\delta})}{[(1-a)(\rho+\overline{\delta})+a\overline{\delta}(1+\chi)](1+\theta)}$. Notice that as $\theta > 0$, $\overline{\phi}^0 > \underline{\phi}^0$ holds. Moreover, the local concavity condition in this case becomes $\sigma \geq 0$.

The positive determinant implies that

$$\phi > 1 - \frac{(1+\chi)(\rho+\overline{\delta})}{(1-a)(1+\theta)(\rho+\overline{\delta}) + a(1+\theta)(\rho+\overline{\delta})(1+\chi)}(> \overline{\phi}^0).$$

Therefore, local indeterminacy cannot exist.

(ii) When $\gamma = 1$, the trace in this KPR case becomes

$$\mathcal{T} = \frac{1}{\Delta}\{\rho(1-\phi)\beta + \alpha[\frac{\rho+\overline{\delta}}{a} - \overline{\delta}(1-\phi)][\frac{-\sigma(1+\chi)Q(1)}{\chi+1-Q(1)} - \sigma\chi - \sigma]\}.$$

Therefore, $\overline{\rho}^1 > 0$ exists such that the trace is negative if and only if $\rho \in (0, \overline{\rho}^1)$ and $\phi \in (\max(0, \underline{\phi}^1), \min(\overline{\phi}^1, 1))$ holds, where $\underline{\phi}^1$ and $\overline{\phi}^1$ are the two roots of the polynomial[1]

$$(1+\chi)\mathcal{C}^2(\phi) - \mathcal{C}(\phi)\mathcal{B} + \mathcal{D} = 0, \tag{A17}$$

where

$$\mathcal{B} = \{(1+\chi)[a + a(1+\theta)] + (1-a)(1+\theta)\frac{\rho+\overline{\delta}}{\overline{\delta}} + (\frac{1}{\sigma} - 2)(1-a)\frac{\rho+\overline{\delta}}{\overline{\delta}}\},$$

$$\mathcal{D} = \{(1+\chi)a^2(1+\theta) + (1-a)a(1+\theta)\frac{\rho+\overline{\delta}}{\overline{\delta}} + (\frac{1}{\sigma} - 2)(1-a)\frac{\rho+\overline{\delta}}{\overline{\delta}}a(1+\theta)\},$$

and

$$\mathcal{C}(\phi) = \frac{\rho+\overline{\delta}}{\overline{\delta}(1-\phi)}.$$

Notice that as $\mathcal{B}^2 - 4(1+\chi)\mathcal{D} > 0$, $\overline{\phi}^1 > \underline{\phi}^1$ holds. This requires that

$$\mathcal{A}_1^2\theta^2 + [2\mathcal{A}_1^2 + 2\mathcal{A}_1\mathcal{B}_1 - 4(1+\chi)\mathcal{D}_1]\theta + [(\mathcal{A}_1 + \mathcal{B}_1)^2 - 4(1+\chi)\mathcal{D}_1] > 0,$$

where $\mathcal{A}_1 = (1+\chi)a + (1-a)\frac{\rho+\overline{\delta}}{\overline{\delta}}$, $\mathcal{B}_1 = (1+\chi)a + (\frac{1}{\sigma} - 2)(1-a)\frac{\rho+\overline{\delta}}{\overline{\delta}}$, and $\mathcal{D}_1 = \{(1+\chi)a^2 + (1-a)a\frac{\rho+\overline{\delta}}{\overline{\delta}} + (\frac{1}{\sigma} - 2)(1-a)\frac{\rho+\overline{\delta}}{\overline{\delta}}a\}$. In other words, we require that $\theta > \underline{\theta}^1 \geq 0$. Moreover, the local concavity condition in this KPR case becomes $\sigma \geq \sigma_1(\phi) = \frac{(1+\chi)Q(1)}{(1+\chi)Q(1)+Q(1)+\chi(1+\chi)}$, where $Q(1) = \frac{(1+\chi)(1-a)(\rho+\overline{\delta})}{[\frac{\rho+\overline{\delta}}{(1-\phi)} - \overline{\delta}a](1+\chi)+(1-a)(\rho+\overline{\delta})}$. Because $\sigma_1(\phi)$ is decreasing in ϕ, $\sigma_1(\phi) \in (\sigma_1(\min(\overline{\phi}^1, 1)), \sigma_1(\max(0, \underline{\phi}^1)))$.

The positive determinant implies that

$$\phi > \frac{\rho+\overline{\delta}}{\overline{\delta}}[1 - \frac{1}{a(1+\theta)}].$$

Therefore, local indeterminacy in the KPR case exists if and only if $\sigma \geq \sigma_1(\phi)$, $\rho \in (0, \overline{\rho}^1)$, $\theta \in (\underline{\theta}^1, +\infty)$, and $\phi \in (\max(0, \underline{\phi}^1), \min(\overline{\phi}^1, 1))$.

By the continuity property, critical values $\underline{\gamma} \in (0,1)$ and $\overline{\gamma} \in (\underline{\gamma}, 1]$ exist such that for any $\gamma \in (\underline{\gamma}, \overline{\gamma}]$, $\underline{\phi} \in [0,1)$, $\overline{\phi} \in (\underline{\phi}, 1)$, $\overline{\rho} \in (0, +\infty)$, and $\underline{\theta} \in (0, +\infty)$ exist. Local indeterminacy arises if and only if $\rho \in (0, \overline{\rho})$, $\theta \in$

[1]When ρ is very small, $\mathcal{T} < 0$ implies that $\Delta > 0$. The same argument can be seen in Abad et al. (2017).

$(\underline{\theta}, +\infty)$, and $\phi \in (\underline{\phi}, \overline{\phi})$. And the prerequisite is that the local concavity condition holds.

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