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Product Variety and Wage Inequality

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This paper analyzes how product variety affects skilled-unskilled wage inequality. Through building several two-sector general equilibrium models, we find that an increase in product variety will expand wage inequality. This shows that the evolution of the product markets will generate a force to enlarge wage inequality.

Key Words: Product variety; Skilled-unskilled wage inequality; General equilibrium model.

JEL Classification Numbers: J31, L20, O15.

1. INTRODUCTION

Rising skilled-unskilled wage inequality is an important phenomenon that has been deeply investigated from different perspectives. The existing studies can be roughly divided into four strands. The first strand of literature stresses the role of trade and international factor mobility in influencing wage inequality (e.g., Wood, 1995; Marjit et al., 2004; Zhu and Trefler, 2005; Verhoogen, 2008; Oladi et al., 2011; Beladi et al., 2008, 2011, 2013; Pi et al., 2013; Barua and Pant, 2014). The second strand of literature highlights the role of technological change (e.g., skill-biased technological change) in affecting wage inequality (e.g., Autor et al., 1998; Berman et al., 1998; Acemoglu, 2002; Card and DiNardo 2002; Haskel and Slaughter 2002; Oladi and Beladi 2008; Pi and Zhang 2018a). The third strand of literature emphasizes the role of governmental behavior and institutional arrangements in impacting wage inequality (e.g., Chaudhuri and Yabuuchi, 2007; Mandal and Marjit, 2010; Pi and Zhou, 2013; Pi and Chen, 2016; Pi and Zhang, 2016, 2017, 2018b; Chaudhuri et al., 2018; Pi and Fan, 2019a,

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2019b). The fourth strand of literature underlines the role of firm and market behavior in changing wage inequality (e.g., Wälde and Weiß, 2007; Anwar, 2009, 2013; Zhang, 2012; Pi and Zhang 2018c). This paper is most related to the fourth strand. Although there are many studies exploring skilled-unskilled wage inequality in the background of product variety and providing the mechanism on the basis of product variety (e.g., Gupta and Dutta, 2012, 2013; Dutta et al., 2013; Dutta, 2014; Zhang, 2013), they do not analyze how product variety affects skilled-unskilled wage inequality. This paper tries to fill the current research gap.

Product variety has caught more and more attention from economists.[1] According to Lancaster (1990, p. 189), "The term product variety is being used here to refer to the number of variants within a specific product group, corresponding broadly to the number of 'brands' as the term is used in the marketing literature or the number of 'models' in consumer durable markets." There are three lines of literature in this direction that need to be reviewed. The first line is about how product variety is determined from the perspective of market structure (e.g., Lancaster, 1975; Dixit and Stiglitz, 1977; White, 1977; Caminal, 2016, 2019). Just as Lancaster (1990, p. 189) points out, "Economists primarily interested in market structure theory will tend to emphasize competitive relationships, product differentiation and product variety as decision variables for the firm, and the types of market equilibria that result." The second line is about the evolution of product variety (e.g., Swann, 1990; Uzumeri and Sanderson, 1995; Kaniovski, 2005; Dercole et al., 2008). For instance, Swann (1990) analyzes the evolution of product variety on the basis of the process of product competition and the competitive environment. Uzumeri and Sanderson (1995) analyze the forces and the rate of change that influence the evolution of product variety. Kaniovski (2005) analyzes the role of product variety in an evolutionary selection model. Dercole et al. (2008) analyze how technological branching gives rise to product variety through evolution.[2] The third line is about the impact of product variety (e.g., Fischer and Harrington Jr., 1996; Funke and Ruhwedel, 2001). For example, Fischer and Harrington Jr. (1996) investigate how product variety influences firm agglomeration. Funke and Ruhwedel (2001) explore how product variety affects economic

[1] The literature review on product variety can be referred to Lancaster (1990), Ranaivoson (2005), and Chang (2012).

[2] The difference between the first line and the second line is as follows. The first line treats product variety as a "fast" variable similar to the variables of price and quantity, while the second line treats product variety as a "slow" variable, and treats price and quantity as "fast" variables. In other words, the first line adopts the approach that product variety can be decided by the firm, just as price or quantity can be decided by the firm. However, the second line holds that product variety evolves on the basis of the related environment, and thus can be treated as an exogenous variable when price and quantity are analyzed.

growth. This paper is most related to the second and third lines. However, the existing literature in these three lines neglects to examine the impact of product variety on skilled-unskilled wage inequality. To the best of our knowledge, this paper is the first one to analyze how product variety influences skilled-unskilled wage inequality.

In order to investigate how product variety exerts an impact on skilled-unskilled wage inequality, this paper builds several two-sector general equilibrium models that take both the fixed cost and the variable cost into account. For the two sectors, one sector is monopolistically competitive and non-tradable, and the other sector is perfectly competitive and tradable. We consider three types of cost components of the monopolistically competitive sector. First, we consider the case that this sector uses capital as the fixed input and skilled labor and capital as the variable input. Second, we consider the case that skilled labor and capital are used as both the fixed input and the variable input. Third, we consider the case that this sector employs capital and skilled labor as the fixed input, and capital and unskilled labor as the variable input. The results show that an increase in product variety will surely expand skilled-unskilled wage inequality.

The economic mechanism of this paper is as follows. Incomes of factor owners can be derived from production costs, which include the fixed cost and the variable cost in the monopolistically competitive sector and the production cost in the perfectly competitive sector. With an increase in product variety, the percentage ratio of the fixed cost to the total income will intuitively rise. Consumers are factor owners with Dixit-Stiglitz preferences, and thus spend a certain proportion of their incomes on differentiated products. This implies that the percentage ratio of the variable cost to the total income remains unchanged. Accordingly, the proportion of the production cost in the perfectly competitive sector to the total income will decline. For the case that skilled labor is employed as the variable input and unskilled labor is used in the perfectly competitive sector, the total wage and the wage rate of skilled labor will increase more sharply (or drop more mildly) than the wage rate of unskilled labor. As a result, skilled-unskilled wage inequality will be expanded. The economic mechanism behind the other two cases are similar.

The rest of this paper is organized as follows. Section 2 provides the basic model. Section 3 gives the extended models. Section 4 makes some concluding remarks.

2. THE BASIC MODEL

In this section, we consider an economy with a skilled sector and an unskilled sector. The skilled sector employs skilled labor and capital. The unskilled sector uses unskilled labor and capital as input. Capital is allowed to move freely between the two sectors.

In our assumed economy, the unskilled sector is perfectly competitive and tradable. Its production technology exhibits constant returns to scale. The world price of the unskilled products is normalized to unity, and the cost-minimizing condition of unskilled sector is given by:

$$a_{AU}w_U + a_{AK}r = 1, \tag{1}$$

where w_U is the wage rate of unskilled labor, r is the unit return to capital. a_{AU} and a_{AK} are the amounts of unskilled labor and capital used to produce one unit of unskilled product, respectively.

The skilled sector is monopolistically competitive. The exogenous variable n denotes the firms that provide differentiated products in this sector and it is a measurement of product variety. A representative firm in the skilled sector uses capital as a fixed input and employs skilled labor and capital as a variable input. Following Anwar (2006, 2008), Combes et al. (2008), Zhang (2012, 2013), Pi et al. (2013), and Pi and Zhou (2015), the total cost function of the representative firm is given by:

$$C = fr + uxw_S^{\beta}r^{1-\beta}, \tag{2}$$

where f and u represent the fixed input and the variable input, respectively. x is the output of the representative firm. β is a parameter that lies in $(0, 1)$. w_s is the wage rate of skilled labor.

The factors of production are assumed to be fully employed. Thus, we have:

$$\begin{aligned} \beta nuxw_S^{\beta-1}r^{1-\beta} &= \overline{L}_S, & (3)\\ a_{AU}A &= \overline{L}_U, & (4)\\ nf + (1-\beta)nuxw_S^{\beta}r^{-\beta} + a_{AU}A &= \overline{K}, & (5) \end{aligned}$$

where A denotes the output of the unskilled sector. $\overline{L}_S$, $\overline{L}_U$, and $\overline{K}$ are the factor endowments of skilled labor, unskilled labor, and capital, respectively.

The consumers of differentiated products are factor owners with Dixit-Stiglitz preference (see Dixit and Stiglitz 1977; Fujita et al., 1999). The social utility function and the composite good are respectively given by the following equations:

$$\begin{aligned} U &= M^{1-\theta}A_C^{\theta}, & (6)\\ M &= \left(\int_0^n x_i^{\frac{\sigma-1}{\sigma}}di\right)^{\frac{\sigma}{\sigma-1}}, & (7) \end{aligned}$$

where A_C is the consumption of unskilled products. When the unskilled sector is tradable, the consumption of unskilled products is not necessarily equal to the output of the unskilled sector in our assumed economy. x_i is the output of the ith variety of differentiated products. σ is the elasticity of substitution between two different kinds of differentiated products. θ is a parameter that is between 0 and 1.

It is not difficult to verify that the elasticity of substitution is equal to the price elasticity of demand for each variety of differentiated products. Thus, the profit-maximizing condition of firms in the skilled sector is given by:

$$p\left(1-\frac{1}{\sigma}\right)=uw_S^{\beta}r^{1-\beta}, \tag{8}$$

where p is the endogenous price of differentiated products.

The price of differentiated products is determined by market clearing. Then, we obtain:

$$\frac{1-\theta}{p}(w_S\overline{L}_S+w_U\overline{L}_U+r\overline{K})=nx, \tag{9}$$

where the left-hand side of Eq. (9) represents the demand for skilled products and the right-hand side of Eq. (9) represents the supply.

The basic model is composed of six equations, Eq. (1), Eqs. (3) to (5), and Eqs. (8) to (9). w_S, w_U, r, p, x, and A are six endogenous variables. The only exogenous variable is n. Other variables are parameters. In the existing literature, Dutta et al. (2013), Gupta and Dutta (2013), and Zhang (2013) also treat n as an exogenous variable.[3] Such a treatment can help us to focus on the impact of product variety.

We establish Proposition 1 to describe the impact of product variety on skilled-unskilled wage inequality.

Proposition 1. *When there is an increase in product variety, (i) the wage rate of skilled labor will increase (resp. decrease) if the capital intensity in the unskilled sector is relatively low (resp. high); (ii) the wage rate of unskilled labor will decrease; and (iii) skilled-unskilled wage inequality will be expanded.*

The proof of Proposition 1 is relegated to Appendix A1.

The economic intuition behind Proposition 1 is as follows.

An increase of product variety implies that more capital will be used as the fixed input, and less capital is employed by the incumbents in the skilled sector and the unskilled sector.

[3]Here, it should be noted that Zhang (2013) treats n as an exogenous variable in the short run, and treats n as an endogenous variable in the long run.

To begin our analysis, we consider two assumed economies as benchmarks. First, we consider an economy (denoted as B_1) where the prices of skilled and unskilled products are determined exogenously. Product variety remains unchanged, and there is a capital outflow in this economy. Second, we consider an economy (denoted as B_2) where the price of differentiated products is determined endogenously, and other settings are similar to those of B_1. It is worth noting that we do not care for the usage of capital that is drawn away from the incumbents in the skilled sector and the unskilled sector when considering B_1 and B_2. With the help of B_1 and B_2, we decompose the effect of increasing product variety into three parts: a substitution effect, a mismatch effect and a diversity effect.

For the economy B_1, an outflow of capital leads to an increase in the return of capital r. Faced with rising (i.e., r), both sectors use labor to substitute capital to maintain their initial marginal cost. The productivities of skilled labor and unskilled labor will decrease. With exogenously determined prices, the wage rates will decline. This effect is called the substitution effect in this paper. The change direction of wage inequality depends on the capital intensities of the skilled sector and the unskilled sector, which has been analyzed by Pi and Zhou (2015). When the variable input of the skilled sector is more capital-intensive than the unskilled sector, the output of the skilled sector declines more sharply than the unskilled sector (i.e., $\hat{n} + \hat{x} = \hat{x} = \hat{M} < \hat{A} < 0$) and the wage rate of skilled labor drops more severely than that of unskilled labor. Otherwise, the opposite is true.

For the economy B_2 where the price of differentiated products is determined endogenously, we find that the equilibrium in B_1 is not an equilibrium in B_2. In the case of $\hat{x} < \hat{A} < 0$, the expenditure on differentiated products, which equals the sales income of differentiated products, declines more sharply than the total income. There will be an excessive demand for the differentiated products, which causes an increase in the price of differentiated products (i.e., $\hat{p} > 0$), draws capital from the unskilled sector and then reduces the output of the unskilled sector. However, in the case of $\hat{A} < \hat{x} < 0$, there will be an excessive supply of differentiated products. Then, the price of differentiated products declines and capital flows from the skilled sector to the unskilled sector. The skilled sector shrinks and the output of the unskilled sector increases. This adjustment process continues until $\hat{p} + \hat{x} = \hat{A}$ because there is no excessive demand or supply at this point. The aforementioned adjustment derives from the mismatch between production and consumption, and we call it the mismatch effect. Furthermore, it is not difficult to verify that the sum of the substitution effect and the mismatch effect leads to a decrease in the output of the unskilled sector (i.e., $\hat{A} < 0$).

Finally, we come to the assumed economy depicted in the basic model. When product variety increases, a consumer who loves diversity can promote his economic welfare by rearranging his consumption without increasing his spending on differentiated products. Accordingly, the marginal utility of differentiated products will increase. There will be an excessive demand for differentiated products. Compared with the equilibrium in B_2, the price of differentiated products will rise further and this draws capital from the unskilled sector. Compared with the equilibrium in B_2, the sales income of differentiated products is higher and the unskilled sector shrinks further. This also implies more (resp. less) unskilled products will be imported (resp. exported) in our assumed economy. From the analysis above, there holds $\hat{p} + \hat{x} + \hat{n} > \hat{A}$. This shows that the production value of the skilled sector decreases more mildly than that of the unskilled sector when product variety increases. We call this effect the diversity effect.

In the basic model, unskilled labor is fully employed by the unskilled sector and skilled labor is fully employed by the skilled sector. With diminishing marginal return of unskilled labor, it is not difficult to verify that $\hat{w}_U \leq \hat{A}$. The allocative share of skilled labor is determined by the exogenous parameters σ and β. Finally, we have $\hat{w}_S = \hat{p}+\hat{x}+\hat{n} > \hat{A} > \hat{w}_U$. There is a widened wage gap with increased product variety.

Moreover, we find that the sum of the substitution effect and the mismatch effect results in a decrease in the sales income of differentiated products (recalling that $\hat{p} + \hat{x} = \hat{A}$). Contrarily, the diversity effect contributes to an increase of the sales income. When the unskilled sector is relatively capital-intensive, faced with a given capital outflow, the output of the unskilled sector drops mildly. Then, the diversity effect overwhelms the sum of the substitution effect and the mismatch effect. The sales income of the skilled sector increases and the wage rate of skilled labor also increases. Otherwise, the sum of the substitution effect and the mismatch effect overwhelms the diversity effect, the wage rate of skilled labor decreases.

3. THE EXTENDED MODELS

In this section, we extend the basic model by considering the skilled sector with different types of production technologies.

3.1. Firms Use Unskilled Labor and Capital as the Fixed Input and the Variable Input

In this subsection, we consider the case that skilled labor is also used as the fixed input. The production cost function of a representative firm in the skilled sector is changed to:

$$C = f w_S^{\alpha} r^{1-\alpha} + u x w_S^{\beta} r^{1-\beta}, \tag{10}$$

where α is a parameter between 0 and 1.

Accordingly, Eqs. (3) and (5) are respectively substituted by Eqs. (11) and (12):

$$\alpha n f w_S^{\alpha-1} r^{1-\alpha} + \beta n u x w_S^{\beta-1} r^{1-\beta} = \overline{L}_S, \tag{11}$$

$$(1-\alpha) n f w_S^{\alpha} r^{-\alpha} + (1-\beta) n u x w_S^{\beta} r^{-\beta} + a_{AK} A = \overline{K}. \tag{12}$$

So far, the new model has been built. It contains six equations, i.e., Eq. (1), Eq. (4), Eq. (8), Eq. (9), Eq. (11), and Eq. (12)), and six endogenous variables, i.e., w_S, w_U, r, p, x, and A. n is the only exogenous variable. Other variables are parameters.

We use Proposition 2 to describe how product variety influences skilled-unskilled wage inequality.

PROPOSITION 2. *When product variety increases, (i) if the fixed input becomes relatively more labor-intensive (i.e., $\alpha > \alpha^*$) and/or the unskilled sector becomes relatively more labor-intensive (i.e., $\theta_{AU} > \theta_{AU}^*$), the wage rate of skilled labor will rise; however, if the fixed input becomes relatively less labor-intensive (i.e., $\alpha < \alpha^*$) and/or the unskilled sector becomes relatively less labor-intensive (i.e., $\theta_{AU} < \theta_{AU}^*$), the wage rate of skilled labor will decrease; (ii) the wage rate of unskilled labor will reduce; and (iii) skilled-unskilled wage inequality will be expanded.*

The proof of Proposition 2 is relegated to Appendix A2.

The economic explanation of Proposition 2 is as follows. To begin with, we consider two extreme examples where $\alpha = 0$ and $\alpha = 1$. Unsurprisingly, we find that the results described by Proposition 2 is the same as those in Proposition 1 when $\alpha = 0$.

When $\alpha = 1$, according to the substitution effect, capital flows from the skilled sector to the unskilled sector. The wage rates (i.e., w_S and w_U) and the output of unskilled labor (i.e., A) increase. However, the return of capital (i.e., r) and the output of skilled labor (i.e., nx) decrease. Considering the mismatch effect, there will be an excessive demand of differentiated products, which increases the price of the skilled sector and makes capital flow from the unskilled sector to the skilled sector until $\hat{p} + \hat{x} = \hat{A}$. This contributes to an increase in the capital intensity of the skilled sector and the wage rate of skilled labor rises more sharply than the return to capital during this adjustment. To judge the symbol of $\hat{A}$, we will check whether $\hat{A} = 0$ is an equilibrium. Since all unskilled labor is employed by the unskilled sector, an unchanged A implies that the amount of capital employed by the unskilled sector also remains unchanged. Combined with exogenously determined price of the unskilled product, the wage rate of unskilled labor, as well as the return to capital, does not change. However, the wage rate of skilled labor increases, and this implies that the total income of our assumed economy increases. If there is $\hat{p} + \hat{x} = \hat{A} = 0$, then an excessive demand of differentiated products emerges. This leads to a rise in the price of differentiated products and makes capital flow from the unskilled sector to the skilled sector. So, at equilibrium, $\hat{p} + \hat{x} = \hat{A} < 0$. The sum of the substitution effect and the mismatch effect causes a decrease in w_U. An increase in r and w_S. Accordingly, wage inequality is widened. Intuitively, the more skilled labor is employed as the fixed input, the more severely the wage rate of skilled labor will increase. An excessive demand for differentiated products prevents capital from flowing out of the skilled sector, and draws capital from the unskilled sector. The diversity effect causes a further increase (resp. decrease) in the wage rate of skilled (resp. unskilled) labor. Combining these three effects, we conclude that $\hat{w}_S > \hat{r} > 0 > \hat{w}_U$ in the case of $\alpha = 1$.

When α is a value between 0 and 1, it is not difficult to verify that less capital and skilled labor are employed as the variable input by the skilled sector and the unskilled sector. This situation is just a combination of the cases where $\alpha = 0$ and $\alpha = 1$. α denotes the weights of the aforementioned two situations. Whenever $\alpha = 0$ or $\alpha = 1$, there is a widened wage gap and a declining wage rate of unskilled labor. So, we conclude that similar results will be obtained when α lies between 0 and 1. When it comes to the wage rate of skilled labor, the result is more complicated. When α is relatively big, the fixed input is relatively labor-intensive, and the change direction of skilled labor is much similar to that of $\alpha = 1$ (i.e., $\hat{w}_S > 0$). However, when the fixed input is relatively capital-intensive, the change direction of w_S is ambiguous and determined by the labor intensity of the unskilled sector, which is the conclusion of Proposition 1.

3.2. Firms Use Skilled Labor and Capital as the Fixed Input, and Use Unskilled Labor and Capital as the Variable Input

In this subsection, we explore the case that firms in the skilled sector employs unskilled labor and capital as the variable input.

Accordingly, for a representative firm, its total cost function is given by:

$$C = f w_S^{\alpha} r^{1-\alpha} + u x w_U^{\beta} r^{1-\beta}. \tag{13}$$

Thus, Eqs. (3) to (5) and Eq.(8) will be changed to:

$$\alpha n f w_S^{\alpha-1} r^{1-\alpha} = \overline{L}_S, \tag{14}$$

$$a_{AU}A + \beta n u x w_U^{-1+\beta} r^{1-\beta} = \overline{L}_U, \tag{15}$$

$$(1-\alpha) n f w_S^{\alpha} r^{-\alpha} + (1-\beta) n u x w_U^{\beta} r^{-\beta} + a_{AK}A = \overline{K}, \tag{16}$$

$$p\left(1-\frac{1}{\sigma}\right) = u w_U^{\beta} r^{1-\beta}. \tag{17}$$

So far, a new model containing six equations has been built. Six endogenous variables (i.e., w_S, w_U, r, p, x, and A) are determined by six equations (i.e., Eq. (1), Eq. (9), and Eqs.(14)-(17)). The only exogenous variable is n. Other variables are parameters.

We use Proposition 3 to describe how product variety affects skilled-unskilled wage inequality.

PROPOSITION 3. *When product variety increases, (i) the wage rate of skilled labor will increase; (ii) if the employment scale of unskilled labor in the skilled sector is relatively big (resp. small), i.e.,* $\frac{\lambda_{MU}}{\lambda_{AU}}$ *is relatively big (resp. small), the wage rate of unskilled labor will increase (resp. decrease); and (iii) skilled-unskilled wage inequality will be enlarged.*

The proof of Proposition 3 is relegated to Appendix A3.

The economic intuition of Proposition 3 is straightforward.

When product variety increases, a representative firm in the skilled sector employs $\frac{\overline{L}_S}{n}$ units of skilled labor as the input and this implies that more capital will be utilized as the fixed input. Then, the return of capital increases to draw capital from the variable input and the unskilled sector. The wage rate of skilled labor will increase more severely because differentiated firms employ capital to substitute skilled labor as the fixed input (i.e., $\hat{w}_S > \hat{r} > 0$).

First, we decompose the impact of increasing product variety into two parts. A rise in the return to capital and the wage rate of skilled labor contributes to an increase in total income. Then, there is an excessive demand of differentiated products. The capital intensity of the skilled sector decreases and it implies $\hat{w}_U < \hat{r}$ regardless of the nominal price of differentiated products. Combining with $\hat{w}_S > \hat{r}$, we have $\hat{w}_S > \hat{r} > \hat{w}_U$. Accordingly, the output of the tradable unskilled sector decreases and the wage rate of unskilled labor declines. Then, we conclude that increasing product variety leads to a wider wage gap. Second, the wage rate of unskilled labor employed by the skilled sector increases due to increasing total income. Besides, the wage rate of the unskilled sector, as well as the return to capital, increases. When unskilled labor is mostly distributed in the skilled sector, which is not a common case, the former force dominates the latter, then there will be an increase in the wage rate of unskilled labor. A more common case is that unskilled labor is mostly distributed in the unskilled sector, and in this case the wage rate of unskilled labor will decrease.

4. CONCLUDING REMARKS

This paper investigates the effect of product variety on skilled-unskilled wage inequality. In order to conduct the analysis, this paper builds several general equilibrium models. Our finding is that an increase in product variety will widen wage inequality. This shows that the evolution of the product markets on the basis of product variety will produce a force to enlarge wage inequality. In other words, the product markets themselves have a tendency to amplify wage inequality when product variety is taken into account.

Our theoretical results are associated with the well-known "Dutch Disease" and can be traced back to some classical studies (e.g., Richards, 1994). In the case of Dutch Disease, a boom in exported energy leads to an increase in national wealth. Then the demand for non-tradable products increases. Economic resources are redistributed between sectors and this leads to a squeeze of the tradable sector. As Richards (1994) points out, this analysis could be generalized to any sector facing a limited demand. In our model, an increase in product variety promote demands for differentiated products, which changes the structural composition of the economy and makes the tradable sector squeeze relatively.

In the future research, this paper can be extended in the following directions. Firstly, similar to that mentioned in Pi and Zhou (2015), we can consider the case that production costs of the monopolistically competitive sector are endogenously determined. Secondly, similar to Markusen and Venables (2000), we can incorporate trade into our analytical framework, and may draw some new insights. Thirdly, similar to Caminal (2016, 2019), we can embed game theory into our analytical framework, and strategical interactions may shed some new light on the issue. We hope that this paper can attract more scholars into this promising field.

APPENDIX

Appendix A1: Proof of Proposition 1

Totally differentiating Eq. (1), Eqs. (3) to (5), and Eqs. (8) to (9), we obtain:

$$\begin{pmatrix} -\beta & 0 & -1+\beta & 1 & 0 & 0 \\ 0 & \theta_{AU} & \theta_{AK} & 0 & 0 & 0 \\ \beta\lambda_{MVK} & \theta_{AU}\lambda_{AK}\sigma_A & -\theta_{AU}\lambda_{AK}\sigma_A-\beta\lambda_{MVK} & 0 & \lambda_{MVK} & \lambda_{AK} \\ 0 & -\theta_{AK}\sigma_A & \theta_{AK}\sigma_A & 0 & 0 & 1 \\ -1+\beta & 0 & 1-\beta & 0 & 1 & 0 \\ -\Pi_S & -\Pi_U & -\Pi_K & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} \hat{w}_S \\ \hat{w}_U \\ \hat{r} \\ \hat{p} \\ \hat{x} \\ \hat{A} \end{pmatrix}$$

$$= -\begin{pmatrix} 0 \\ 0 \\ \lambda_{MFK}+\lambda_{MVK} \\ 0 \\ 1 \\ 1 \end{pmatrix}\hat{n} \tag{A.1}$$

where σ_A is the factor substitution elasticity in the unskilled sector. θ_{AU} and θ_{AK} are the distributive shares of unskilled labor and capital used by the unskilled sector. λ_{MFK}, λ_{MVK}, and λ_{AK} denote the allocative shares of capital used as the fixed input in the skilled sector, as the variable input in the skilled sector, and as the input in the unskilled sector, respectively. Π_S, Π_U, and Π_K are the national income shares of skilled labor, unskilled labor, and capital. The symbol "ˆ" denotes the proportional change of a variable (e.g., $\hat{x}=\frac{dx}{x}$). The aforementioned notations are in line with Jones (1965, 1971).

Solving Eq. (A.1), we obtain: $\frac{\hat{w}_S}{\hat{n}}=\frac{\lambda_{MFK}(\theta_{AU}\Pi_K-\theta_{AK}\Pi_U)}{\lambda_{AK}(1-\Pi_S)\sigma_A+\lambda_{MVK}\Pi_U}$, and $\frac{\hat{w}_U}{\hat{n}}=-\frac{\lambda_{MFK}\theta_{AK}(1-\Pi_S)}{\lambda_{AK}\sigma_A(1-\Pi_S)+\lambda_{MVK}\Pi_U}$.

If $\frac{\theta_{AK}}{\theta_{AU}}<\frac{\Pi_K}{\Pi_U}$, then $\frac{\hat{w}_S}{\hat{n}}>0$; and if $\frac{\theta_{AK}}{\theta_{AU}}>\frac{\Pi_K}{\Pi_U}$, then $\frac{\hat{w}_S}{\hat{n}}<0$.

Thus, we have: $\frac{\hat{w}_S-\hat{w}_U}{\hat{n}}=\frac{\lambda_{MFK}\Pi_K}{\lambda_{AK}\sigma_A(1-\Pi_S)+\lambda_{MVK}\Pi_U}>0$.

Appendix A2: Proof of Proposition 2

Totally differentiating Eq. (1), Eq. (4), Eq. (8), Eq. (9), Eq. (11) and Eq (12), we obtain:

$$\begin{pmatrix} -\beta & 0 & -1+\beta & 1 & 0 & 0 \\ 0 & \theta_{AU} & \theta_{AK} & 0 & 0 & 0 \\ \alpha\lambda_{MFK}+\beta\lambda_{MVK} & \theta_{AU}\lambda_{AK}\sigma_A & -\theta_{AU}\lambda_{AK}\sigma_A-\alpha\lambda_{MFK}-\beta\lambda_{MVK} & 0 & \lambda_{MVK} & \lambda_{AK} \\ 0 & -\theta_{AK}\sigma_A & \theta_{AK}\sigma_A & 0 & 0 & 1 \\ -1+\alpha\lambda_{MFS}+\beta\lambda_{MVS} & 0 & 1-\alpha\lambda_{MFS}-\beta\lambda_{MVS} & 0 & \lambda_{MVS} & 0 \\ -\Pi_S & -\Pi_U & -\Pi_K & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} \hat{w}_S \\ \hat{w}_U \\ \hat{r} \\ \hat{p} \\ \hat{x} \\ \hat{A} \end{pmatrix}$$

$$= -\begin{pmatrix} 0 \\ 0 \\ \lambda_{MFK}+\lambda_{MVK} \\ 0 \\ 1 \\ 1 \end{pmatrix} \hat{n} \tag{A.2}$$

where λ_{MFS} and λ_{MVS} represent the allocative shares of skilled labor used as the fixed input and the variable input in the skilled sector, respectively.

Solving Eq. (A.2), we obtain:

$$\frac{\hat{w}_S}{\hat{n}} = \frac{-\psi_1\theta_{AU} + \Pi_U(\lambda_{MFK}\lambda_{MVS} - \lambda_{MVK}\lambda_{MFS}) - \lambda_{AK}\lambda_{MFS}\sigma_A}{\Delta},$$

where $\Delta = \lambda_{AK}(-1+\alpha\lambda_{MFS}+\Pi_S\lambda_{MVS})\sigma_A - \theta_{AK}\Pi_U[\alpha\lambda_{MFK}\lambda_{MVS}+(1-\alpha\lambda_{MFS})\lambda_{MVK}] < 0$, and $\psi_1 = \lambda_{MFK} - \Pi_S(\lambda_{MFK}\lambda_{MVS} - \lambda_{MVK}\lambda_{MFS}) > 0$.

The sign of the term $\Pi_U(\lambda_{MFK}\lambda_{MVS} - \lambda_{MVK}\lambda_{MFS}) - \lambda_{AK}\lambda_{MFS}\sigma_A$ is determined by the parameter α. By simple calculation, we have:

$$\lambda_{MFK}\lambda_{MVS} - \lambda_{MVK}\lambda_{MFS} = \lambda_{MVK}\lambda_{MFS}\left(\frac{1-\alpha}{\alpha}\frac{\beta}{\beta-1} - 1\right). \tag{A.3}$$

Thus, if $\alpha > \alpha^*$, then $\Pi_U(\lambda_{MFK}\lambda_{MVS} - \lambda_{MVK}\lambda_{MFS}) - \lambda_{AK}\lambda_{MFS}\sigma_A \leq 0$ and $\frac{\hat{w}_S}{\hat{n}} > 0$. If $\alpha < \alpha^*$, then the sign of $\frac{\hat{w}_S}{\hat{n}}$ is determined by θ_{AU}. If $\alpha < \alpha^*$ and $\theta_{AU} > \theta^*_{AU}$, then the wage rate of skilled labor will rise. If $\alpha < \alpha^*$ and $\theta_{AU} < \theta^*_{AU}$, then the wage rate of skilled labor will fall. Here, $\alpha^* = \frac{\beta\lambda_{MVK}\lambda_{MFS}\Pi_U}{(1-\beta)\lambda_{AK}\lambda_{MFS}\sigma_A + \lambda_{MVK}\lambda_{MFS}\Pi_U}$, and $\theta^*_{AU} = \frac{1}{\psi_1}[\Pi_U(\lambda_{MFK}\lambda_{MVS} - \lambda_{MVK}\lambda_{MFS}) - \lambda_{AK}\lambda_{MFS}\sigma_A]$.

The related change of w_U is given by:

$$\frac{\hat{w}_U}{\hat{n}} = \frac{\theta_{AK}[\lambda_{MVK}\lambda_{MFS}\Pi_S + \lambda_{MFK}(1-\lambda_{MVS}\Pi_S)]}{\Delta} < 0.$$

Thus, we get:

$$\frac{\hat{w}_S - \hat{w}_U}{\hat{n}} = \frac{-\lambda_{AK}\lambda_{MFS}\sigma_A - \lambda_{MFK} + (\lambda_{MFK}\lambda_{MVS} - \lambda_{MFS}\lambda_{MVK})(\Pi_S + \Pi_U)}{\Delta} > 0.$$

Appendix A3: Proof of Proposition 3

Totally differentiating Eq. (1), Eq. (9), and Eqs. (14)-(17), we obtain:

$$\begin{pmatrix} 0 & -\beta & -1+\beta & 1 & 0 & 0 \\ 0 & \theta_{AU} & \theta_{AK} & 0 & 0 & 0 \\ \alpha\lambda_{MFK} & \theta_{AU}\lambda_{AK}\sigma_A+\beta\lambda_{MVK} & -\theta_{AU}\lambda_{AK}\sigma_A-\alpha\lambda_{MFK}-\beta\lambda_{MVK} & 0 & \lambda_{MVK} & \lambda_{AK} \\ 0 & -\theta_{AK}\lambda_{AU}\sigma_A-(1-\beta)\lambda_{MU} & \theta_{AK}\lambda_{AU}\sigma_A+(1-\beta)\lambda_{MU} & 0 & \lambda_{MU} & \lambda_{AU} \\ -1+\alpha & 0 & 1-\alpha & 0 & 0 & 0 \\ -\Pi_S & -\Pi_U & -\Pi_K & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} \hat{w}_S \\ \hat{w}_U \\ \hat{r} \\ \hat{p} \\ \hat{x} \\ \hat{A} \end{pmatrix}$$

$$= -\begin{pmatrix} 0 \\ 0 \\ \lambda_{MFK}+\lambda_{MVK} \\ \lambda_{MU} \\ 1 \\ 1 \end{pmatrix}\hat{n} \tag{A.4}$$

where λ_{MU} and λ_{AU} are the allocative shares of unskilled labor employed by the skilled sector and the unskilled sector, respectively.

Solving Eq. (A.4), we obtain:

$$\frac{\hat{w}_S}{\hat{n}} = \frac{\lambda_{AK}\lambda_{AU}\sigma_A + \lambda_{AU}\psi_2 + \lambda_{AK}\lambda_{MU}(\Pi_K + \theta_{AK}\Pi_S)}{(1-\alpha)[\lambda_{AK}\lambda_{AU}\sigma_A + \lambda_{AK}\lambda_{MU}(\Pi_K+\Pi_S) + \lambda_{AU}\lambda_{MVK}\Pi_U]} > 0,$$

where $\psi_2 = \theta_{AU}\lambda_{MFK} + \theta_{AU}\lambda_{MVK}\Pi_S + \lambda_{MVK}\Pi_U > 0$;

$$\frac{\hat{w}_U}{\hat{n}} = \frac{\theta_{AK}[\lambda_{AK}\lambda_M U\Pi_S - \lambda_{AU}(\lambda_{MFK} + \lambda_{MVK}\Pi_S)]}{(1-\alpha)[\lambda_{AK}\lambda_{AU}\sigma_A + \lambda_{AK}\lambda_{MU}(\Pi_K+\Pi_S) + \lambda_{AU}\lambda_{MVK}\Pi_U]}.$$

If $\frac{\lambda_{MU}}{\lambda_{AU}} > \lambda^*$, then $\frac{\hat{w}_S}{\hat{n}} > 0$; and if $\frac{\lambda_{MU}}{\lambda_{AU}} < \lambda^*$, then $\frac{\hat{w}_U}{\hat{n}} < 0$. Here, $\lambda^* = frac\lambda_{MFK} + \lambda_{MVK}\Pi_S\lambda_{MVK}\Pi_S > 1$.

Thus, we obtain:

$$\frac{\hat{w}_S - \hat{w}_U}{\hat{n}} = \frac{\lambda_{AK}(\lambda_{AU}\sigma_A + \lambda_{MU}\Pi_K) + \lambda_{AU}[\lambda_{MFK} + \lambda_{MVK}(\Pi_S + \Pi_U)]}{(1-\alpha)[\lambda_{AK}\lambda_{AU}\sigma_A + \lambda_{AK}\lambda_{MU}(\Pi_K+\Pi_S) + \lambda_{AU}\lambda_{MVK}\Pi_U]} > 0.$$

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