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Duality in an Industry with Fluctuating Demand

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A perfect-competition model is developed to analyze duality in specialization and technology such as in the men's clothing industry, an industry with highly seasonal nature of the business cycle. We show that when the market fluctuation is large enough, some firms will specialize in one variety with the advantage of static efficiency, while other firms will generalize in multi-variety production as a means of self-insurance. The specialized firms mainly satisfy the stable component of market demand, while the generalized firms satisfy only the variable components of demand. Relative to the specialized firms, the generalized firms have a smaller firm size and a lower degree of vertical division of labor within the firm, and use the technology with more flexible specialization but less capital-labor ratio.

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1. INTRODUCTION

Since the second half of the nineteen century, the traditional mode of craft production has been substituted little by little by the modern mode of mass production. The mode of mass production has mainly the following characteristics: internalizing the production process (vertical integration), dividing the production into higher degrees of specialization, using machines with advanced and product-specific technology, having a larger firm size and a higher capital-labor ratio, etc.

However, a large number of facts show that mass production does not replace other production modes in all industries. In some industries with

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the characteristics of a large number of items and a small scale [Scott 1983a, 1983b, 1984 and 1996, Huys et al. 1999, etc] craft production or flexible specialization[1] dominates. In other industries, especially in those with very unstable demand [Berger and Piore 1980, Sable 1982, Fraser 1983, Piore and Sabel 1984, Hiebert 1990], small-scale production coexists with mass production, According to Piore and Sabel's theory, the production in such an industry is divided into two sectors. The first sector mainly satisfies the stable component of the market — the bottom part of the business cycle, which is suitable to achieving the static efficiency of mass production. On the other hand, the secondary sector tends to be more flexible. It uses less sophisticated product-specific technology and arranges the production with a lower degree of division of labor to satisfy the variable component of demand. This is the duality referred to in this paper.

By the end of the first decade of the twentieth century, for example, the men's clothing industry in USA was segmented according to the size of industrial unit, product lines, and market share. The unstable portion of the market comprised a large share of mass market for cheaper ready-to-wear clothing and was increasingly served by a large mass of small producers. The more predictable component of the market was largely restricted to medium- and higher-priced lines produced by a handful of large, technically sophisticated firms [Fraser 1983 p531]. In addition to the size and technology, they show duality in internal vertical division of labor. In Chicago in 1900s, for example, not counting the production of raw materials and machines, the "inside" plants took sixty workers to produce a coat, fifty to make a pair of pants, twenty a vest, and eighty to eighty-five to produce a single overcoat [Fraser p543]. On the other hand, small shops, assigned a given number of garments as their expected output, introduced instead a simpler form of group production known as the task team, which usually consisted of a baster, a sewing machine operator, a finisher, and a presser [Fraser p534]. Similar to Fraser's case study, Hiebert [1990] shows that fluctuating demand is an important cause of the duality in garment production in Toronto during the period between 1915 and 1931.

The purpose of this paper is to develop a model to analyze duality such as in the clothing industry. We begin providing a model of vertical division of labor. By changing some basic assumptions and comparing the results, we uncover some causalities which are not easily seen from case studies.

First, duality theory emphasizes the duality in machine usage: the specialized firms use machines with advanced technology while the generalized firms use simple machines (tools) in production. However, our model shows that the duality in machine usage is not a necessary condition of duality.

[1] Flexible specialization is thought of "as the manufacture of a wide and changing array of customized products using flexible, general purpose machinery and skilled, adapted labor" [Hirst and Zeitlin 1991].

For simplicity we assume that all firms do not use machines (or use the same machines) in production. On the one hand, the higher the degree of vertical division of labor in the production chain, the higher productivity is achieved. On the other hand, the vertical division of labor needs to be reorganized from the production of one variety to another. The higher degree of vertical division of labor requires more time to reorganize in shifting from the production of on variety to another. Therefore, the specialized firms prefer a higher degree of vertical division of labor while the generalized firms (which produce more varieties) prefer less. When machines are taken into account, the difference between special-purpose machines and flexible machines enriches the duality in an industry, says the dualities in the productivity of machines and in the capital-labor ratios.

Next, though in reality duality involves ex-ante heterogeneous firms, we show that duality may emerge among ex-ante identical firms. Duality arises from the fact that some firms gain from static efficiency by applying mass production, while the others benefit from flexibility by applying flexible specialization. The ex-ante heterogeneity may help the firms to choose the mode of production, but it is not a necessary condition of duality.

Mills and Schumann [1985] show that, in a fluctuating market, duality will exist when some of the firms use more variable inputs instead of fixed inputs to increase flexibility. Firms using this strategy will have a smaller firm size and a lower capital-output ratio. In reality, flexible specialization and substitution of variable inputs for fixed inputs may be used in a fluctuating market. However, there are some differences between these two flexible strategies. First, the degrees of flexibility in these two strategies are different. With flexible specialization a firm has higher flexibility (i.e. lower transfer costs) in multi-variety production, while from substituting of variable inputs for fixed inputs a firm has higher output-flexibility (i.e. lower opportunity costs in varying output). Second, the duality may occur in ex-ante identical firms which apply flexible specialization, while in their model the existence of duality requires a discrete set of cost options. Without this additional condition, the duality does not exist in ex-ante identical firms [Sheshinski and Dreze 1976, Mills 1984]. Last, capital usage is not a necessary condition of duality by applying flexible specialization, but it is a necessary condition in their model. If all inputs are variable, there is no duality in their model. Besides, their model does not discuss the vertical division of labor.

This paper is organized as follows. Section 2 presents the basic assumptions of this paper. Section 3 provides an equilibrium analysis, and applications and discussions are undertaken in Section 4. This paper concludes in section 5, and the proofs of all propositions are in section 6.

2. THE BASIC ASSUMPTIONS

2.1. The market demand

For representing the highly seasonal nature of the business cycle in the clothing industry, we assume that there are two varieties, X and Y, and their market demand functions are, respectively

$$x = MK_x^\theta/p_x^\theta, \quad \theta > 0 \tag{1a}$$
$$y = Mk_y^\theta/p_y^\theta, \quad \theta > 0 \tag{1b}$$

where x (y) is the quantity demanded and p_x (p_y) is the price of the market X (Y), M is an index of the extent of the market demand, and θ is the absolute elasticity of market demand. For simplicity, we assume that indexes k_x and k_y are symmetrically distributed as:

$$(k_x, k_y) = \begin{cases} (1+k, 1-k), & \text{with probability} 0.5 \\ (1-k, 1+k), & \text{with probability} 0.5 \end{cases} \tag{2}$$

where k $(0k < 1)$ is an index of the degree of the market fluctuation. Relative to the situation of $k_x = 1 - k$, the market demand for X is higher when $k_x = 1 + k$. According to the definition of Berger and Piore [1980, p66], the total market demand is divided into a stable component $\{M[(1+k)^\theta - (1-k)^\theta]/p_x^\theta\}$ and a variable component $[M(1-k)^\theta/p_x^\theta]$.

2.2. The vertical chain of production

Due to the symmetry of products X and Y, in the later part of this paper, we will only present the production of X; the production of Y is similar.

We use the interval $(0, 1]$ to denote the infinitely divisible vertical chain in the production process of the final goods X. For example, $X_{(s,s']}$ denotes the production process from intermediate goods s to intermediate goods s', where $0 < s < s' < 1$; $X_{(0,s']}$ denotes the production process from the beginning to intermediate goods s' where $s' < 1$, or to final goods where $s' = 1$; and $X_{(s,1]}$ denotes the production process from intermediate goods s to final goods, where $0 < s < 1$. For convenience we use $X_{s'}$ to denote $X_{(0,s']}$, where $0 < s' < 1$, and use X to denote $X_{(0,1]}$.

Vertical division of labor within a firm means that the interval $(0, 1]$ is divided into some independent intervals, and a worker just works in parts of the intervals but no one covers the whole interval $(0, 1]$. If a worker works in some intervals, we would say the worker specializes in these intervals.

2.3. Production functions

Three kinds of inputs are required in production. The first is labor, we assume that workers are identical, and each is endowed with one unit of time for production. The second is machines. For representing the difference in the technology of machines, we denote t ($t > 0$) as the productivity of a machine. In addition, we assume that each worker is equipped with one machine. This assumption is consistent with reality in the clothing industry: sewing machines, pressing machines, the machines for buttonhole making and others are the main physical capitals; usually each machine is used by one worker. The third is intermediate goods. When a person produces in interval $(s, s']$, intermediate goods X_s is required.

Denote $(s, s']$ as one piece of specialized task in the process of producing X. We assume that $X_{(s,s']}$ has the Leontief production function:

$$X_{(s,s']} = \min\left\{X_s, \frac{t[l-(s'-s)b]}{s'-s}\right\} \tag{3}$$

where X_s is the amount of intermediate input at the stage s, l is the production time, t is the productivity of machines, and $(s'-s)b$ is the entry costs. Though partly used for simplicity, the assumption of a Leontief production function is not unacceptable for the problem here. Thus, we would need a certain amount of flour/cloth to make a certain amount of bread/garments.

Different from the definition of a production function in standard economic theory, we begin the definition from one piece of specialized task. As shown in Lemma 1 below, different arrangements in the division of labor will give rise to different production functions.

Let us explain the production functions of (3). Assume that X has an increasing-returns production function where there is no division of labor in the vertical production chain:

$$x = t(l-b) \tag{4}$$

where b is the entry cost and hence $l-b$ is the effective production time; since t is the productivity coefficient, thus $t(l-b)$ amount of output is produced. Moreover, we assume that there is a symmetric structure in the production process of X which implies the following two points.

First, since the production of X requires the entry cost b, under the condition that there is no economies (or diseconomies) of scope, we need only $(s'-s)b$ as the entry cost in the production process of $X_{(s,s']}$, and hence $l-(s'-s)b$ is the effective production time.

Second, assume that one final good consists symmetrically of two components, component one and component two, and assume that using machines with productivity t, one unit of effective production time can produce t units of final goods. Thus one unit of effective production time can produce $2t$ units of either component one or component two. It means that as the scope of the production decreases, the productivity increases accordingly. From (3) we know that one unit effective production time can produce t units of $X_{(0,1]}$. As the production interval is changed from $(0,1]$ to $(s,s']$, from symmetry, one unit of effective production time can produce $t/(s'-s)$ units of $X_{(s,s']}$. Hence, using machines with productivity t, the effective production time $l-(s'-s)b$ can potentially produce

$$X_{(s,s']} = \frac{t[l-(s'-s)b]}{s'-s} \tag{5}$$

Thus, as a Leontief production function, the intermediate goods and the labor contribution will satisfy (3).

Now consider the arrangement of division of labor for output maximization. Suppose that there are n workers in a firm with production technique (3), where each worker is equipped with a machine with productivity t and devotes l units of working time to production.

FIG. 1.

LEMMA 1. *The arrangement for output maximization is that* $(0,1]$ *is evenly divided into* n *independent subintervals (see Figure 1), where each worker produces in one subinterval. The firm's maximum output level is*

$$X = t(nl-b) \tag{6}$$

Lemma 1 shows that the production exhibits the economies of vertical specialization (i.e. the productive efficiency under division of labor is higher than that without division of labor). In the case of no division of labor, all people do the same task. According (3), the output is

$$X = nt(l-b) \tag{7}$$

Comparing (6) with (7), the production line with vertical division of labor has a higher level of productivity due to the fact that each worker needs to pay a lower amount of entry cost. The workers just need to learn the skill in a sector in the production line instead of learning the skill of the whole product.

2.4. Putty-clay technology

There is trade-off between productivity and flexibility, involving two aspects. The first is the arrangement of vertical division of labor in a production line. As shown in Lemma 1, the vertical division of labor in a production line increases the productivity by reducing each worker's entry cost. However, it needs the workers in the production line to spend time for coordination. The more workers vertically specializing in a production line, the more coordination time each worker spends. Besides, shifting from producing one variety to another, say from jean to skirt, they need to reorganize the division of labor. Thus, the more varieties they produce, the more coordination time each worker needs to spend. Therefore, relative to the specialized firms, the small shops, which mainly produce the custom-made products, prefer less degree of vertical division of labor. On the other hand, from craft production to mass production, machines are developed to increase productivity but at the cost of flexibility [Faunce 1965]. The higher the productivity of a machine, more time is required to transfer from producing one variety to another.

According to this idea, in a firm which produces m varieties in a production line with n vertically specialized workers and n machines with productivity t used respectively by each worker, we assume that each worker needs to pay $f(m)tn^{\alpha}$ unit of coordination time in production, where $\alpha > 0$ and f is an increasing function. It reveals that the coordination time is positively correlated with the number of varieties, the degree of vertical division of labor, and the productivity of the machines[2].

Since X and Y are different varieties of the same product, for simplicity we assume that they share the same entry cost. That means if a worker specializes in subinterval $(s, s']$, according to production function (3), she just needs to pay an entry cost of $(s' - s)b$, no matter which varieties she produces. This assumption is consistent with the situation in the clothing industry. It requires almost the same knowledge in a specialized sector, such as cutting, sewing, buttonhole making, or pressing.

2.5. The factor markets

In this model, three kinds of factors are taken into account. The first is labor. We assume that people are identical and each individual is endowed with one unit of time for production.

The second is machines. We assume that the price of machines with productivity t is ct^2, where c is the machine cost coefficient. Our idea is

[2] Becker and Murphy [1992] develop a model to study the vertical division of labor in a cooperative team. Vertical division of labor increases the productivity in the one hand but coordinative cost in the other. The optimal division of labor is decided by the trade-off between the marginal productivity and the marginal coordinative cost. But their model does not relate to multi-variety production and machine usage.

that, from the aspect of technology, machines with different productivity levels are available. Which level of machines is used is determined by relative economic factors. Here, the cost coefficient c is an inverse index representing the technical level in the machine industry. The lower the level of c, the more advanced the machine industry is, and hence the machines with higher productivity are used. To avoid the possibility of machines with infinite productivity, we assume that $\alpha > 1$.

The third is financing. In reality both hiring employees and purchasing machines need financing, but the costs of financing are different. Comparatively purchasing machines requires a much longer period in financing. For simplicity, we assume that it does not need any financing for employment, while purchasing machines needs financing available at interest rate r. For example, the cost of purchase a machine with productivity t is rct^2, where ct^2 is the price of the machine.

For simplicity, we assume that the factor markets are perfect competition where the wage rate, interest rate, and machine cost coefficient are exogenous variables. It implies that relative to the market demand of final goods, the elasticities of supply for all kinds of factors are infinite.

3. THE MODELS

3.1. The general model

As a standard assumption of long-run equilibrium, we assume that the firms in the model are free to enter or exit any industry. It implies that the maximum (expected) profit the firms earn is zero, and the prices in equilibrium relate to the minimum average costs of the firms but not to the market demands.

We assume that the firms are risk neutral and maximize (expected) profits. Since vertical specialization is considered, the firms in this model have several variables to choose: (1) the number of employees (n); (2) the vertical division of labor within the firm; according to Lemma 1, the even division of the interval $(0, 1]$ where each worker produces in one subinterval is optimal for maximum output; (3) the productivity level of the machines (t).

In this model, a firm may specialize in producing X or Y, or generalize in producing both X and Y. First, we discuss about the specialized firms in variety X.

Denote p_h as the price in higher demand (when $k_x = 1 + k$) and p_l the price in lower demand (when $k_x = 1 - k$) of X, thus $(0.5p_h + 0.5p_l)$ is the expected price of output, where the prices are determined endogenously. When production chain $(0, 1]$ is divided evenly into n pieces of subintervals, each worker needs to pay the entry cost (b/n) and the coordination time $[f(1)tn^\alpha]$, thus her effective production time is $[1 - f(1)tn^\alpha - b/n]$.

According to Lemma 1 the firm's output is $n - f(1)tn^{\alpha+1} - b$. Since n workers are hired, the firm has to pay wages amounting to wn. Next, the firm has to pay the costs of n pieces of machines (rct^2n), where ct^2 is the price of a machine with productivity t and r is the interest rate. Thus the firm's expected profit maximization is represented as:

$$E\pi = \max\{(0.5p_h + 0.5p_l)(n - f(1)tn^{\alpha+1} - b)t - rct^2n - wn\} \ \ st: n > 0, t > 0 \tag{8}$$

On the other hand, a firm may generalize in producing two varieties as a means of self-insurance [Ehrlich and Becker 1972], i.e. she may produce the variety when it has a higher market demand.

Because of the assumption of symmetry in X and Y, a firm will use half of the production time ($0.5n$) in producing X when $k_x = 1+k$, and the other half of the time in producing Y when $k_y = 1+k$. According to Lemma 1, she has the same output $0.5[n - f(2)tn^{\alpha+1} - b]$ in X and Y, where $f(2)tn^{+1}$ represents the sum of coordination time associated with the twice vertical division of labor within the firm, once for producing in X and the other for Y. From the symmetry of X and Y, the two varieties have the same higher price (p_h). Relative to the specialized firm, the benefit a generalized firm gets is that she always faces with the higher price (p_h), but at the cost of increasing the coordination time from $f(1)t_sn_s^{\alpha+1}$ to $f(2)t_gn_g^{\alpha+1}$. Thus the profit maximization of a generalized firm is represented as:

$$\pi = \max\left\{2p_h\left(\frac{n - f(2)tn^{\alpha+1} - b}{2}\right) - rct^2n - wn\right\} \tag{9}$$

Proposition 1. *There is a unique long-run equilibrium in this model in which we have two situations.*

(1)In the situation that half of the firms specialize in variety X and the other half specialize in variety Y, the equilibrium variables $(n_s, t_s, p_s, p_{h1}, p_{l1})$ in variety X (Y) satisfy (10) to (14), respectively.

$$\frac{[\alpha(n_s - b) - 2b]n_s^{2\alpha+2}}{n_s - n} = \frac{\alpha rcb^2}{f(1)^2w} \tag{10}$$

$$t_s = \sqrt{\frac{w[\alpha(n_s - b) - 2b]}{rc\alpha(n_s - b)}} \tag{11}$$

$$p_s = \frac{2brc}{f(1)n_s^{\alpha}[\alpha(n_s - b) - 2b]} \tag{12}$$

$$p_{h1} = (1+k)p_s \tag{13}$$

$$p_{l1} = (1-k)p_s \tag{14}$$

(2)In the situation that some firms specialize in X or Y, other firms generalize in producing X and Y. For the firms specializing in X (Y), the equi-

librium variables (n_s, t_s) *satisfy (10) and (11). On the other hand, for the firms generalizing in* X *and* Y*, the equilibrium variables* $(n_g, t_g, p_g, p_{h2}, p_{l2})$ *satisfy (15) to (18), respectively.*

$$\frac{[\alpha(n_g - b) - 2b]n_g^{2\alpha+2}}{n_g - b} = \frac{\alpha r c b^2}{f(2)^2 w} \tag{15}$$

$$t_g = \sqrt{\frac{w[\alpha(n_g - b) - 2b]}{rc\alpha(n_g - b)}} \tag{16}$$

$$p_{h2} = p_g = \frac{2brc}{f(2)n_g^{\alpha}[\alpha(n_g - b) - 2b]} \tag{17}$$

$$p_{l2} = 2p_s - p_g \tag{18}$$

(3)The equilibrium occurs in situation (1) [(2)] if and only if (19a) [(19b)] is satisfied.

$$p_{h1} \leq p_s \tag{19a}$$

$$p_{h_1} > p_s \tag{19b}$$

where $n_s(n_g)$ *is the degree of vertical division of labor,* $t_s(t_g)$ *is the productivity of the machines,* $p_s(p_g)$ *is the expected price which makes zero expected profit, and the price* $p_h(p_l)$ *in higher (lower) demand, the indexes with subscript* g *and* s *denote the indexes of the generalized firms and specialized firms, respectively.*

According to Proposition 1, there are two situations in equilibrium. First, half of the firms will specialize in variety X and the other half will specialize in variety Y. In this situation, the expected profit of each firm is zero. A firm gets positive profits when the market demand is higher and negative profits when the market demand is lower. But why no firm would generalize in producing two varieties? It is because that a firm will not gain from the change from specializing in one variety to generalizing in two varieties [see (19a)]. From (12), (13) and (17) we know that this situation occurs because the market fluctuation is small enough (less k) and/or the increase in coordination time associated with the multi-variety production is large enough (larger $f(2) - f(1)$)]. In this situation, though a generalized firm can always face the higher price (p_h), the price p_h, which is positively correlated with the degree of fluctuation, is not large enough to overcome the increase in coordinative costs. Thus no firm will generalize in producing two varieties.

In contrast, when the degree of fluctuation is large enough, and/or the increase in coordinative costs is small enough, some firms will earn more profits from the change from specializing in one variety to generalizing in two varieties [see (19b)]. The generalized firms will increase the market supply when the market demand is higher, and thus reduces the degree of fluctuation in these two varieties. Therefore, the more firms generalize in two varieties, the lower the degree of fluctuation in these varieties, and hence the lower the increase in profit from this change. It continues until the generalized firms earn the same amount of profit as the specialized firms do. In long-run equilibrium the profits earned by both the specialized and generalized firms equal zero.

3.2. The simple model

In this simple model we only take labor into account, but ignore machines as inputs. We will show that the dualistic structures will occur in an economy without machines. Though in reality the usage of machines is closely related to this phenomenon, it is not a necessary condition.

Accordingly, we modify some assumptions. First, we assume that the productivity of a worker equals one (i.e. $t = 1$). Second, in a firm with n workers vertically specialized in production, each worker needs to pay $f(1)n^{\alpha}(f(2)n^{\alpha})$ units of coordination time in a specialized (generalized) firm.

Similar to the general model, the expected profit maximization of a specialized firm is represented as:

$$E\pi = \max\{(0.5p_h + 0.5p_l)(n - f(1)n^{\alpha+1} - b) - wn\} \quad st: n > 0 \tag{20}$$

On the other hand, the profit maximization of a generalized firm is represented as:

$$\pi = \max\left\{2p_h\left(\frac{n - f(2)n^{\alpha+1} - b}{2}\right) - wn\right\} \quad st: n > 0 \tag{21}$$

Corollary 1. *There is a unique long-run equilibrium in this model in which we have two situations.*

$$(1+k)[\alpha^{\frac{\alpha}{\alpha+1}} - (\alpha+1)b^{\alpha\alpha+1}f(2)^{\frac{1}{\alpha+1}}] \leq \alpha^{\frac{\alpha}{\alpha+1}} - (\alpha+1)b^{\frac{\alpha}{\alpha+1}}f(1)^{\frac{1}{\alpha+1}} \tag{22a}$$
$$(1+k)[\alpha^{\frac{\alpha}{\alpha+1}} - (\alpha+1)b^{\frac{\alpha}{\alpha+1}}f(2)^{\frac{1}{\alpha+1}}] > \alpha^{\frac{\alpha}{\alpha+1}} - (\alpha+1)b^{\frac{\alpha}{\alpha+1}}f(1)^{\frac{1}{\alpha+1}} \tag{22b}$$

(1) When (22a) is satisfied, half of the firms will specialize in variety X and the other half will specialize in variety Y, but no firm will generalize in two varieties. The equilibrium variables $(n_s, p_s, p_{h1}, p_{l1})$ in variety X (Y)

satisfy (23) to (26), respectively.

$$n_s = \left[\frac{b}{\alpha f(1)}\right]^{\frac{1}{\alpha+1}} \tag{23}$$

$$p_s = \frac{w\alpha^{\frac{\alpha}{\alpha+1}}}{\alpha^{\frac{\alpha}{\alpha+1}} - (\alpha+1)b^{\frac{\alpha}{\alpha+1}} f(1)^{\frac{1}{\alpha+1}}} \tag{24}$$

$$p_{h1} = (1+k)p_s \tag{25}$$

$$p_{l1} = (1-k)p_s \tag{26}$$

(2)When (22b) is satisfied, some firms will specialize in X or Y, other firms will generalize in producing X and Y. For the firms specializing in X (Y), the equilibrium variables (n_s) satisfy (23). On the other hand, for the firms generalizing in X and Y, the equilibrium variables (n_g, p_g, p_{h2}, p_{l2}) satisfy (27), (28) and (29), respectively.

$$n_g = \left[\frac{b}{\alpha f(2)}\right]^{\frac{1}{\alpha+1}} \tag{27}$$

$$p_{h2} = p_g = \frac{w\alpha^{\frac{\alpha}{\alpha+1}}}{\alpha^{\frac{\alpha}{\alpha+1}} - (\alpha+1)b^{\frac{\alpha}{\alpha+1}} f(2)^{\frac{1}{\alpha+1}}} \tag{28}$$

$$\begin{aligned} p_{l2} &= 2p_s - p_g \\ &= \frac{2w\alpha^{\frac{\alpha}{\alpha+1}}}{\alpha^{\frac{\alpha}{\alpha+1}} - (\alpha+1)b^{\frac{\alpha}{\alpha+1}} f(1)^{\frac{1}{\alpha+1}}} - \frac{w\alpha^{\frac{\alpha}{\alpha+1}}}{\alpha^{\frac{\alpha}{\alpha+1}} - (\alpha+1)b^{\frac{\alpha}{\alpha+1}} f(2)^{\frac{1}{\alpha+1}}} \end{aligned} \tag{29}$$

where $n_s(n_g)$ is the degree of vertical division of labor, $p_s(p_g)$ is the expected price which yields zero expected profit, and the price $p_h(p_l)$ in higher (lower) demand, the indexes with subscript g and s denote the indexes of the generalized firms and specialized firms, respectively.

Corollary 1 shows the existence of duality in an economy without machines. When the degree of fluctuation (k) is large enough, and/or the increase in coordination time $[f(2) - f(1)]$ is small enough [i.e. when (22b) is satisfied], the dualistic structures will occur in the unique long-run equilibrium. Relative to the specialized firms, the generalized firms have lower degrees of vertical division of labor and firm size.

4. APPLICATIONS AND DISCUSSIONS

In this section we will explain the duality in men's clothing industry [Fraser 1983] in terms of our model. For the representation of the economic structure, two endogenous variables are mainly used in this model.

First, the degree of vertical division of labor within the firm is indexed by the number of people (n) working in the vertical production chain of the firm, which is also the index of the size of the firm.

The next is the productivity of machines used in the firm (t). Since we assume that each person is endowed with one unit of labor time and uses one machine, the capital-labor ratio equals the price of the machines divided by wage rate (ct^2/w). Thus the larger the productivity of the machines used by the firm, the larger the capital-labor ratio the firm has.

4.1. The duality

Fraser [1983] shows the duality in specialization and technology in men's clothing industry. Relative to the specialized firms, which apply mass production to satisfy the stable component of market demand, the generalized firms apply flexible specialization to satisfy the variable components of demands with a smaller firm size, a lower degree of vertical division of labor within the firm, and a lower capital-labor ratio.

Corollary 2. *Comparing the firms which generalize in producing two varieties with the firms which specialize in producing one variety, we get:*

(1)A firm specializing in one variety has a larger degree of vertical division of labor within the firm and a larger firm size ($n_g < n_s$).

(2)A firm specializing in one variety uses the machines with a higher productivity level and the technology with a higher capital-labor ratio in production ($t_g < t_s$).

where the indexes with subscript g and s denote the indexes of the generalized firms and specialized firms, respectively.

The nature of Corollary 2 is the trade-off between productivity and flexibility, which represents in the aspects of specialization and technology.

The first aspect is the vertical division of labor within the firm. On the one hand, the higher the degree of vertical division of labor in the production chain, the smaller the production segment and hence the less skill each worker is required in production, thus the higher productivity is achieved. On the other hand, the vertical division of labor requires costs/time for shifting from the production of one variety to another, says, from jeans to coat. In this case, the higher the degree of vertical division of labor, the more the amount of time is required to transfer in multi-variety production. For flexibility, therefore, the generalized firms prefer to arrange the production with a lower degree of vertical division of labor within the firm. Moreover, the more varieties to be produced, the lower the degree of vertical division of labor the firm prefers. This explains why smaller firms, which usually engage in custom-made production, have lower degrees of vertical division of labor.

The analysis above shows that the machines used by the firm are not the necessary conditions of duality in vertical division of labor within the firm. However, they enrich the duality. In the case of men' clothing industry, two kinds of technologies are used: the tools used in craft production and the machines used in mass production. From craft production to mass production, the machines' development is summarized as: (1) the productivities increase gradually; (2) relative to simple machines (tools), the advanced machines use more product-specialization technology, which means that, higher productivity is achieved partly at the cost of flexibility; and (3) the prices of machines are positively correlated to their productivities. Relative to the specialized firms, the generalized firms prefer to use more flexible machines[3], which accordingly have lower productivities and lower prices, and hence the generalized firms have lower capital-labor ratios.

In this model, since we assume that all firms are identical before production, we miss some reasons of duality. In the case of men's clothing industry, For example, the owners of the "small shops" are usually new immigrations from East Europe. They are familiar with the handicraft production taking place in an environment of intense competition among impoverished workers [Fraser p528-29].

4.2. The impact of dualistic structure on fluctuation

Now the question is whether the application of flexible specialization can reduce or even eliminate the market fluctuation? First, the application of flexible specialization can reduce the market fluctuation (indexed by $p_h - p_l$). The generalized firms will increase the market supply when the market demand is higher, and thus reduces the degree of fluctuation in these two varieties. Next, the market fluctuation is eliminated (i.e. $p_h = p_l$) if and only if the transfer cost associated with multi-variety production is zero $[f(2) = f(1)]$. Image that if the fluctuation is digested completely, it means that p_h equals p_l, and hence the specialized firms face the same price as the generalized firms do. Since these two kinds of firms should have the same amount of profit in equilibrium due to their being identical before production, they should have the same amount of costs in this situation. Compared with the specialized firms, the generalized firms require one more kind of costs, the transfer cost. It means that the fluctuation can be digested completely if and only if the transfer cost is zero. Thus we have Corollary 3.

[3]If we assume that all machines have the same flexibility, i.e. in vertical division of labor a worker's coordination time is assumed as $f(m)n^{\alpha}$ instead of $f(m)n^{\alpha}t$, we conclude that all firms use the same machines. It means that the duality in machines is due to the difference in the flexibility of machines.

Corollary 3. *Under the conditions of Proposition 2, we get:*

(1)The market fluctuation is reduced when flexible specialization is applied.

(2)The market fluctuation is eliminated if and only if the transfer cost associated with multi-variety production is zero.

This Corollary offers a point of view to understand accurately a basic argument in duality theory. Berger and Piore [1980 p66-67] argue that under the duality the specialized firms will produce in the stable component of the market demand, while the generalized firms in variable component. Next, if fluctuations are predictable, the specialized firms will produce not only in stable component but also in part of variable component of the market demand Since they emphasize not enough on the second conclusion, people believe that the division of the market by stable and variable components is clear-cut or at least a rough line exists between the two kinds of economic organizations [Fraser p524]. But it is not true. We show that the specialized firms still face fluctuation when the transfer cost is larger than zero. It means that the specialized firms will face both the stable and the variable components of the market demand. The larger the transfer cost, the larger the market fluctuation and hence the larger the variable component the specialized firm will face. The fluctuation is eliminated only when the transfer cost is zero, in this situation the generalized firms digest completely the market fluctuation, i.e. the generalized firms produce only in the variable component, while the specialized firms produce only in the stable component of the market.

Although we emphasize that the specialized firms will generally produce for the stable and variable components of the market, the conclusions about the duality in an industry are still true. It is because the specialized firms will use the mode of mass production for the stable component of the market, though the arrangement will be adjusted for the variable component of the market.

4.3. Two kinds of flexible strategies

Besides the application of flexible specialization, the firms may use more variable inputs instead of fixed inputs to increase the flexibility in a fluctuating market. For profit maximization, a competitive firm will vary its output according to the price of the good. From substituting variable inputs for fixed inputs, a firm has more output-flexibility responding to the market price. As a result, a firm will use a higher proportion of variable inputs in a more fluctuating market.

However, the degrees of flexibility in these two strategies are different. With flexible specialization a firm has higher flexibility (i.e. a lower transfer cost) in multi-variety production, while from using a higher proportion of variable inputs a firm has higher output-flexibility (i.e. less opportunity cost in varying output). Now the question is how these two strategies are cooperatively used in reducing market risks?

As mention above, except for the case where the transfer cost is zero, the application of flexible specialization does not eliminate the fluctuation completely. It means that the specialized firms still face some market fluctuation. In this case, the specialized firms may increase their profits from using a higher proportion of variable inputs in production. Similarly, except the case where all the inputs are variable, the later strategy does not eliminate the market fluctuation. Thus it may still have spaces for some firms to apply flexible specialization in multi-variety production. In this model, since vertical division of labor is considered, employees are not variable inputs, but quasi-fixed inputs. It is because in a firm with internal vertical division of labor, workers need to learn the knowledge (entry cost) in their segment. If a firm dismisses some employees when market demand declines, the workers who keep the job need to produce in a wider range of segment and accordingly need to learn more knowledge. It means that changing the number of workers involves not only dismissing some employees but also increasing the human capital of kept workers. For simplicity, we assume that the entry cost (b) is large enough that the firms prefer not to change the division of labor, i.e. the workers are regarded as fixed inputs. Thus in our model the firms can only apply flexible specialization in fluctuating market.

Since the two strategies use flexible technologies, usually they lead to similar changes in a fluctuating industry. Mills and Schumann [1985] show that, for example, the firms using higher proportion of variable inputs have smaller firm sizes and lower capital- output ratios. Similarly, Corollary 2 shows that the firms applying flexible specialization have a smaller firm size and a lower capital-labor ratio. However, the two strategies show different properties in some aspects.

For example, the duality may exist in ex-ante identical firms when flexible specialization is applied (see Proposition 1). This is because the specialized firms optimize in one industry while the generalized firms in more than one industry. However, there is no duality in ex-ante identical firms by using higher proportion of variable inputs. The optimal inputs chosen by the identical firms are the same. This is because the optimal solution is unique under the assumption that the inputs are incomplete substitutes. For the existence of duality, Mills and Schumann [1985] assume that the firms have discrete sets of cost options. The reason is that in the situation where the optimal solution is unique in a continuum set, there may be more than one optimal solution when the choice is constrained to a discrete subset.

Next, in the case of men's clothing industry, using a higher proportion of variable inputs or/and applying flexible specialization implies that simpler machines are used in a firm. Since we assume that a worker uses one machine, it means that a firm will have less capital-labor ratio when any of these flexible strategies is used. But the conclusion is not so sure nowadays. Catering for the trend of individualization and diversification in products, machines with numerical technology have been widely used since 1970s. They are flexible in multi-variety production through adjusting the program. Besides, they are more expensive than the machines used in mass production. Relative to the specialized firms, therefore, the generalized firms which use these machines will have larger capital- labor ratios.

4.4. The institutions/policies for market stabilization

Since a necessary condition of the application of flexible specialization is that the market fluctuation is large enough, now the question is, will some institutions/policies for market stabilization eliminate the application of this technology?

First, the application of flexible specialization is due to the structural fluctuation in an industry. The policy of counter business cycles may reduce the fluctuation in the aggregative economy, but has not much effect on the structural fluctuation in an industry. Therefore, the importance of flexible specialization is unlikely to have become less important. Next, the application of flexible specialization is due to the price fluctuation in an industry. Though insurance can even out the fluctuation in the revenue of the firms, it cannot reduce the price fluctuation in a market (indexed by $p_h - p_l$). Thus it does not change the situation where some firms may gain from generalizing into two varieties.

However, some factors do affect the applicability of flexible specialization. For example, a means for market stabilization is storing the products in lower market demand but supplying them in higher demand. If the product storage is costless, market fluctuation is eliminated, and hence the technology of flexible specialization will not be applied. In contrast, if flexible specialization is so efficient that the transfer cost associated with multi-variety production is zero, it will eliminate market fluctuation and hence product storage is not necessary. In reality both methods have costs which will be taken into account in the choice of the methods. For example, the duality in men's clothing industry implies that product storage, as a means for market stabilization, has larger costs. Besides the storage costs, it seems that whether the clothing of a style will still be fashionable in the future is one of the important factors considered.

5. CONCLUSION

According to the idea of duality theory, we develop a model to explain the duality in men's clothing industry. We show that when the market fluctuation is large enough, some firms will specialize in one variety with the advantage of static efficiency, while other firms will generalize in multi-variety production as a means of self-insurance. The specialized firms mainly satisfy the stable component of market demand, while the generalized firms satisfy only the variable components of demands. Relative to the specialized firms, the generalized firms have a smaller firm size and a lower degree of vertical division of labor within the firm, and use the technology with more flexible- specialization but a lower capital-labor ratio.

Moreover, we uncover some causalities which is not easily seen from the case study. First, duality theory emphasizes the duality in machines: the specialized firms use machines with advanced technology while the generalized firms use simple machines (tools) in production. However, our model shows that machine usage is not a necessary condition of duality. For simplicity we assume that all firms do not use machines (or use the same machines) in production. On the one hand, the higher the degree of vertical division of labor in the production chain, the higher the achieved level of productivity. On the other hand, the vertical division of labor requires reorganization time for shifting from the production of one variety to another. The higher degree of vertical division of labor requires more time to reorganize. Therefore, the specialized firms prefer a higher degree of vertical division of labor while the generalized firms prefer a lower degree. When machines are taken into account, the difference between special-purpose machines and flexible machines enriches the duality in an industry, including the dualities in the productivity of machines and in the capital-labor ratio.

Next, though in reality duality is related to ex-ante heterogeneous firms, we show that duality may occur among ex-ante identical firms. The duality comes from that some firms gain from the static efficiency in using mass production, while the others from flexibility by applying flexible specialization. The ex-ante heterogeneity may help the firms to choose the mode of production, but it is not a necessary condition of duality.

Although focusing on men's clothing industry, our model is easy to be extended to analyze duality in any industry with fluctuating demand. For this purpose two special assumptions may have to be modified. First, according to the industry of men's clothing, we assume that a worker uses one machine. Besides, compared with the simple machines used in craft production and the advanced machines in mass production, we assume that the productivity of a machine is negatively correlated to its flexibility.

Nevertheless, our model misses some causes of duality. For example, according to Hiebert's [1990] case study, besides fluctuation, the duality in people's preference, i.e. some people prefer inexpensive standardized clothing while others prefer individualized clothing, is a factor of duality in clothing industry. The standardized products are suitable to be produced by large-scale and vertical integrated firms, while individualized products are better produced by a large number of small-scale and vertical disintegrated firms organized through subcontracts and agglomeration.

APPENDIX

Proof of Lemma 1:

Suppose that interval $(0,1]$ is divided evenly into n pieces of independent intervals, according production function (3), worker 1's output is

$$X_1 = A\left(\frac{l}{s_1 - s_0} - b\right)$$

and worker i's output $(2in)$ is

$$X_i = \min\left\{X_{i-1}, A\left(\frac{l}{s_i - s_{i-1}} - b\right)\right\}$$

where $X_n = X$. Thus we have the firm's output

$$X = \min_{1\le i\le n}\left\{A\left(\frac{l}{s_i - s_{i-1}} - b\right)\right\}$$

Thus the arrangement for output maximization is that $(0,1]$ is evenly divided into n independent subintervals $(s_i - s_{i-1} = 1/n)$, where each worker produces in one subinterval. The firm's maximum output level is

$$X = t(nl - b)$$

Proof of Proposition 1:

(1) In an economy where there are only specialized firms, a firm may specialize in producing X or Y, her expected profit maximization is represented as:

$$E\pi = \max\{(0.5p_h + 0.5p_l)(n - f(1)tn^{\alpha+1} - b)t - rct^2n - wn\} \;\; st: n > 0, t > 0$$

Denote $p = 0.5p_h + 0.5p_l$. The first-order condition is

$$p[1 - (\alpha+1)f(1)n^{\alpha}]t - rct^2 - w = 0 \quad \text{(A.1)}$$

$$p(n - 2f(1)tn^{\alpha+1} - b) - 2rctn = 0 \quad \text{(A.2)}$$

The second-order matrix is

$$\pi'' = \begin{pmatrix} \pi_{nn} & \pi_{nt} \\ \pi_{tn} & \pi_{tt} \end{pmatrix} = \begin{pmatrix} -p\alpha(\alpha+1)f(1)t^2n^{\alpha-1} & p-2(\alpha+1)f(1)tn^{\alpha}-2rct \\ p-2(\alpha+1)f(1)tn^{\alpha}-2rc & -2pf(1)n^{\alpha+1}-2rcn \end{pmatrix}$$

Under the condition that a firm has nonnegative profit ($\pi \geq 0$), the second-order condition is satisfied. In long-run equilibrium the maximum profit of a firm is zero, thus

$$p(n - f(1)tn^{\alpha+1} - b)t = rct^2n + wn \tag{A.3}$$

From (A.1), (A.2) and (A.3) we have the degree of vertical division of labor (n_s), the productivity of the machines (t_s), and the expected price (p_s) which makes zero expected profit:

$$\begin{aligned} \frac{[\alpha(n_s-b)-2b]n_s^{2\alpha+2}}{n_s-b} &= \frac{\alpha rcb^2}{f(1)^2w} \\ t_s &= \sqrt{\frac{w[\alpha(n_s-b)-2b]}{rc\alpha(n_s-b)}} \\ p_s &= \frac{2brc}{f(1)n_s^{\alpha}[\alpha(n_s-b)-2b]} \end{aligned}$$

From demand function (1), the price $p_{h1}(p_{l1})$ in higher (lower) demand satisfies:

$$\begin{aligned} p_{h1} &= (1+k)p_s \\ p_{l1} &= (1-k)p_s \end{aligned}$$

(2) In an economy where both specialized and generalized firms exist, the degree of vertical division of labor (n_s) and the productivity of the machines (t_s) in a specialized firm, the expected price (p_s) which makes a specialized firm zero expected profit will satisfy solutions in situation (1).

On the other hand, the profit maximization of a generalized firm is represented as:

$$\pi = \max\left\{2p_h\left(\frac{n-f(2)tn^{\alpha+1}-b}{2}\right) - rct^2n - wn\right\} \quad st: n>0, t>0$$

Similar to the proof in situation (1), we have the degree of vertical division of labor (n_g), the productivity of the machines (t_g), and the expected price (p_g) which makes zero expected profit:

$$\frac{[\alpha(n_g-b)-2b]n_g^{2\alpha+2}}{n_g-b}=\frac{\alpha rcb^2}{f(2)^2 w}$$
$$t_g=\sqrt{\frac{w[\alpha(n_g-b)-2b]}{rc\alpha(n_g-b)}}$$
$$p_{h2}=p_s=\frac{2brc}{f(2)n_g^{\alpha}[\alpha(n_g-b)-2b]}$$

Since we have $p_s=0.5p_h+0.5p_l$, thus the price in lower demand satisfies:

$$p_{l2}=2p_s-p_{h2}=2p_s-p_g$$

(3) When $p_{h1}\le p_g$, a firm will not gain from the change from the specialization in a variety to the generalization in two varieties. Thus the long-run equilibrium occurs in situation (1). On the other hand, when $p_{h1}>p_g$, some firms will gain from the change from the specialization in a variety to the generalization in two varieties. Thus the long-run equilibrium occurs in situation (2).

Proof of Corollary 1:

Corollary 1 is a special case of Proposition 1, we ignore the proof here.

Proof of Corollary 2:

Denote $g(n)=\dfrac{[\alpha(n-b)-2b]n^{2\alpha+2}}{n-b}$

Since and $f(1)\le f(2)$ and $g(n)$ is an increasing function of n, compared (10) with (15) we have $n_g\le n_s$. Next, since we have ngns, from (11) and (16) we have $t_g\le t_s$.

Proof of Corollary 3:

(1) In an economy where there are only specialized firms, from (13) and (14) we have

$$p_{h2}-p_{l2}=2p_s \tag{A.4}$$

While in an economy where both specialized and generalized firms exist, from (17) and (18) we have

$$p_{h2}-p_{l2}=2p_s-p_g \tag{A.5}$$

Compared (A.4) with (A.5) we have the conclusion.

(2) The market fluctuation is eliminated if and only if $p_{h2} = p_{l2}$. From (12), (17) and (18), $p_{h2} = p_{l2}$ if and only if $f(1) = f(2)$, it means the transfer cost associated with multi- variety production is zero.

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