Minimum Wages and Unemployment Benefits in a Unionised Economy:
A Game-Theoretic Approach

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This paper aims at contributing to the labour market effects of minimum wages and unemployment benefits from a game-theoretic viewpoint. In a dual labour market model, the first sector outcome is characterised by bargaining between unions and firms, while in the second sector firms have to pay a statutory minimum wage. The model shows that the effects of minimum wages differ from those of unemployment benefits. Moreover, we show that the labour market outcome depends on the underlying game-theoretic bargaining solution. That is, the labour market effects of unemployment benefits and minimum wages in the Nash bargaining solution differ substantially from the effects if bargaining follows the Kalai-Smorodinsky solution.

Key Words: Nash solution; Kalai-Smorodinsky solution; Union bargaining; Minimum wages; Unemployment benefits
JEL Classification Numbers: C71, J51.

1. INTRODUCTION

Many countries have reformed their labour market policies and institutions in recent years. The two most important institutional factors in most countries are unemployment benefits and minimum wages. Particularly the consequences of minimum wages have been widely and controversially discussed among economists and policymakers. Common wisdom suggested that minimum wages lead to higher unemployment rates. This consensus, however, was challenged by the studies of Card & Krueger (1994, 1995).

These authors gather information on the employment effects of minimum wage hikes with a focus on the US state New Jersey. In 1992, the minimum wage in New Jersey increased from $ 4.25 to $ 5.05. At the same time, the minimum wage in the adjacent state of Pennsylvania remained at $ 4.25. Card and Krueger asked employers in 410 fast food restaurants whether they will lay off workers in response to the higher minimum wage. Based on their responses, the authors state that the higher minimum wage had no significant effect on employment.

These findings received widespread attention and started to change the opinion within the economics profession. A survey by Fuller & Geide-Stevenson (2003) reports that in 1990, 62% of academic economists in the USA agreed with the statement, "a minimum wage increases unemployment among young and unskilled workers", 19.5% partly agreed while 17.5% disagreed. One decade later, these numbers have changed significantly. Now only 46% of respondents agreed while 28% partly agreed and 27% disagreed.

As a response to Card & Krueger (1994, 1995), a large body of literature provides mixed evidence about the minimum wage effects on employment. Considerable studies include, amongst others, Card & Krueger (2000), Deere et al. (1995), Dolado et al. (1996), Kennan (1995), Neumark & Wascher (2000, 2006), Portugal & Cardoso (2006). A recent study by König & Möller (2009) examines employment effects of minimum wages in Germany. They study the effect of coverage extension of wage agreements in construction that was introduced in 1997. While König & Möller (2009) found no significant effects for Western Germany, they show that the coverage extension led to small negative employment effects in Eastern Germany. Metcalf (2007) finds similar results for the UK. He states that there is little or no evidence of any employment effects due to the British national minimum wage. Neumark & Wascher (2006) review evidence from a large number of minimum wage studies and rehashed the academic discussion. Altogether, the empirical evidence on employment effects of minimum wages is not conclusive. Although a majority finds higher unemployment due to minimum wage increases, there is a significant number of studies concluding no employment effects or even positive ones. Thus, there seems to be a need for theoretical models explaining the fact that higher minimum wages may lead to higher employment.

Simple textbook analysis predicts that introducing a wage floor above the equilibrium wage in a competitive labour market causes unemployment. Under this assumption, there is no room for a theoretical explanation of an employment increase. More comprehensive theoretical contributions build on the monopsony model (Stigler 1946), where it is theoretically possible that a minimum wage may boost employment. Alternative approaches introduce efficiency wages (Jones 1987, Manning 1995, Rebitzer & Tay-

lor 1995) or search and matching models (Berg 2003, Flinn 2006, Masters 1999, Swinnerton 1996) into a minimum wage framework. However, the impact of minimum wages in unionised labour markets has rarely been analysed.[1] This gap in the literature seems to be surprising since both minimum wages and union bargaining power coexist in many labour markets. On the one hand, there are explicit minimum wages e. g. in the U.S. and in 20 EU member states. Furthermore, implicit minimum wages exist due to unemployment benefits or other social security systems. On the other hand, union bargaining power is still high particularly in Europe. The average of collective bargaining coverage in the OECD is 64 %, and is even higher in the EU (OECD 2004). That is, almost two-thirds of salaried workers are subject to union-negotiated terms of employment. The interaction between union bargaining, minimum wages and unemployment benefits is thus highly relevant for many labour markets. This paper aims at combining these facts in a general model framework.

The second focus of the paper is to shed some light on the specific bargaining framework. In order to model union wage bargaining, labour economic theory has emphasised the solution concept proposed by Nash (1950). Other bargaining solutions, e. g. the approach by Kalai & Smorodinsky (1975), have been mostly neglected although they exhibit some interesting features. The axiomatic approach of Kalai & Smorodinsky (1975) builds on the criticism of some of Nash's axioms, especially the independence of irrelevant alternatives. Kalai & Smorodinsky (1975) replace this axiom with the property of individual monotonicity and prove that there is only one bargaining rule satisfying their axioms. Their solution consists of equalizing the parties' sacrifice relative to the maximum benefit they can expect. Although McDonald & Solow (1981) considered both the Nash and the Kalai-Smorodinsky (henceforth KS) solutions, subsequent work on wage bargaining has ignored the latter one. One possible reason to explain this ignorance is the difficulty of doing comparative statics, whereas the Nash approach exhibits mathematical convenience and a well-known game-theoretic foundation (Binmore et al. 1986). However, this ignorance seems hard to defend mainly because of two reasons. First, there exists a game-theoretic foundation by Moulin (1984) implementing the KS bargaining solution in a non-cooperative game. Second, current bargaining situations can be often described by mutual relative concessions of, say, a union and a firm (Economist 2002). Furthermore, economic and psychological experiments provide evidence for the view that people compare relative payoffs (Nydegger & Owen 1974, Roth & Malouf 1979). The Nash approach cannot capture this stylised fact because of the independence of

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[1]The most notable exception is Cahuc et al. (2001). To some extent, Cardona & Sáchez-Losada (2006) and Roberts et al. (2000) also deal with the topic.

irrelevant alternatives axiom. Replacing this axiom by the monotonicity axiom–as it has been done by Kalai & Smorodinsky (1975)–allows individuals to compare relative payoffs and is thus in line with the experimental evidence (Alexander 1992).

The paper addresses some important game-theoretic differences between both solutions. We particularly focus on the economic implications of both the parties' outside option and the maximal feasible gains within a union-firm bargaining framework. The only notable exceptions in the previous literature applying the KS solution to union bargaining are Alexander (1992) and Gerber & Upmann (2006). Alexander (1992) shows in a right-to-manage approach that the Nash and the KS solution can lead to quantitatively different wage and employment levels. However, neither the different comparative statics of both solutions nor the role of the outside option are discussed. Gerber & Upmann (2006) study the KS solution in an efficient bargaining framework and highlight some comparative statics properties in partial equilibrium. Their model points out that a higher outside option has positive effects on the bargained wage and negative effects on employment if bargaining follows the Nash solution. However, an increase in the outside option have ambiguous employment effects in the KS solution. While these general results are interesting from a theoretical viewpoint, Gerber & Upmann (2006) do not focus on the economic characteristics of the outside option and the maximal feasible gains in detail. Moreover, the outside option is treated as exogenous variable in their approach.

This paper adds to the literature by implementing both the Nash and the KS solution in a general model of the labour market. Thereby, we focus on the specific economic characteristics of the maximal feasible gain and the outside option. The paper addresses these points using a dual labour market model with a unionised and a minimum wage sector. Bargaining in the unionised sector is analysed under both the Nash and the KS solution, whereas the union's outside option in the bargaining is a weighted average of the minimum wage and unemployment benefit. We show that the labour market outcome as well as the policy conclusions derived from the model not only depend on the underlying bargaining approach but also on the economic interpretation of the bargaining components.

Applying the Nash solution yields quite clear policy implications. Higher unemployment benefit raises union's outside option as well as the union wage and decreases employment in both sectors. A higher minimum wage yields the same result, at least under some specific assumptions on labour market elasticities. However, these results are not as straightforward if bargaining follows the KS solution. Then the wage and employment effects are ambiguous and depend on the specific form of firm's production and worker's utility functions. Hence, both bargaining approaches might imply diametrical policy implications.

The paper is organised as follows. The basic framework of the model is outlined in section 2. Section 3 analyses the wage and employment determination under the Nash and the KS solution. The comparative static results of raising the minimum wage and unemployment benefits are discussed in section 4, while section 5 contains final remarks.

2. THE STRUCTURE OF THE ECONOMY

2.1. Firms

We consider an economy with a dual labour market. Wage and employment in the first sector are determined by bargaining between unions and firms. Firms in the second sector must pay a statutory minimum wage.[2] There are λ homogeneous firms and the same number of unions in the unionised sector, while the number of firms in the minimum wage sector is normalised to unity. Let N denote the available workforce per firm in the union sector and $Z \equiv \lambda N$ the total number of workers in the economy. The production technology of a representative firm in the unionised sector can be described by the function $f(L)$, while production in the minimum wage sector is characterised by $g(M)$. Both production functions obey the usual concavity conditions, i.e. $f'(L) > 0$, $f''(L) < 0$, $g'(M) > 0$ and $g''(M) < 0$, while $L \leq N$ and M denote employment per firm, respectively. All firms sell their output in a competitive goods market, where the output price is normalised to unity.[3] Hence, the profit of a representative firm in the unionised sector can be written as

$$\Pi = f(L) - wL, \tag{1}$$

with w denoting the bargained wage.

The second sector is a competitive labour market where firms have to pay a statutory minimum wage $\tilde{w}$. Employment is adjusted according to the marginal productivity condition

$$M = g'^{-1}(\tilde{w}). \tag{2}$$

Unemployment arises since the minimum wage is assumed to be binding. Since overall employment in the first sector is given by λL, the rate of

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[2]Therewith we cover the stylised fact that in many labour markets sectors with and without union coverage coexist with minimum wage sectors.

[3]In order to avoid difficulties resulting from imperfect competition in the product market, one can think of firms in the two sectors as operating in a world market where prices are taken as exogenous (Sanfey 1993).

unemployment in the whole economy, φ, can be written as

$$\varphi \equiv 1 - \frac{\lambda L + M}{Z}. \tag{3}$$

2.2. Unions and Workers

Each worker inelastically supplies one unit of labour. The union covers all workers at the firm-level and maximises the (expected) utility of a representative member:[4]

$$U(w, L) = lu(w) + (1 - l)u(\bar{w}), \tag{4}$$

where $u(.)$ characterises the state-independent individual utility function, with $u'(.) > 0$ and $u''(.) < 0$. With probability $l \equiv \frac{L}{N}$, a union member can find a job in the first sector, while the probability of getting an alternative income $\bar{w}$ is given by $1 - l$. The utility from this alternative income, $u(\bar{w})$, equals the utility of getting a job in the minimum wage sector or becoming unemployed.[5] The alternative income is exogenous from each union's viewpoint.[6] However, in general equilibrium it is endogenised to close the model.

The utility from the alternative income can be expressed by

$$u(\bar{w}) = pu(\tilde{w}) + (1 - p)u(b), \tag{5}$$

with $p = \frac{M}{Z - \sum_{i=1}^{\lambda} L_i}$ denoting the probability of getting a job in the second sector, and $b < \tilde{w}$ denoting unemployment benefit.

3. WAGE AND EMPLOYMENT DETERMINATION

3.1. Nash Solution

We first analyse the bargaining outcome of the generalised Nash solution (Nash 1950, Binmore et al. 1986). The firm and the union are assumed to bargain simultaneously over both the wage and the level of employment (McDonald & Solow 1981). Empirical evidence for such an efficient

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[4]This formulation has been popular in recent theoretical work on union bargaining (Booth 1995). Empirical evidence is provided, amongst others, by Pencavel (1991).

[5]One could also think of integrating job search into the outside option. That is, when a union member does not get a job at his union's firm, he searches for a job elsewhere in the unionised sector. Since we focus on the differences between Nash and KS, however, this formulation would not change our results qualitatively.

[6]This assumption follows from the decentralised bargaining approach. Since there is a large number of union-firm pairs, neither a union nor a firm has any impact on the alternative income.

bargaining can be found, amongst others, in MaCurdy & Pencavel (1986) and Svejnar (1986). MaCurdy & Pencavel (1986) show for the American newspaper industry and its primary labour union, the International Typographical Union, that the efficient bargaining model comes closer to reality. Svejnar (1986) states for the U.S. industry that for many firms and unions the outcome can be better characterised by efficient bargaining.

Wage and employment are chosen as to maximise the weighted product of each party's net return from reaching an agreement. The general maximisation problem can be written as

$$\max_{w,L} \Omega = \delta \ln \left[U - \bar{U} \right] + (1 - \delta) \ln \left[\Pi - \bar{\Pi} \right], \tag{6}$$

where $\delta \in [0,1]$ denotes union's bargaining power. The union's outside option in the bargaining $\left(\bar{U} \right)$ is exogenous. However, in general equilibrium the outside option is determined endogenously and then given by (5). Firm's profit in the case when bargain breaks down $\left(\bar{\Pi} \right)$ is assumed to be zero. Hence, the Nash maximand can be written as

$$\max_{w,L} \Omega = \delta \ln \left[l \left(u(w) - u\left(\bar{w} \right) \right) \right] + (1 - \delta) \ln \left[f(L) - wL \right]. \tag{7}$$

We derive the labour market equilibrium from the first-order conditions of the maximisation problem (7). Differentiating (7) with respect to w and L and reformulating yields the contract curve and the rent-division curve.

The contract curve describes the Pareto-efficient outcomes:

$$w = f'(L) + \frac{u(w) - u\left(\bar{w} \right)}{u'(w)}. \tag{8}$$

Due to worker's risk aversion, the right-hand side of (8) increases in w. The slope of the contract curve (8) has a positive sign and the bargained wage exceeds the marginal product of labour.

The rent-division curve

$$w = \delta \frac{f(L)}{L} + (1 - \delta) f'(L) \tag{9}$$

indicates that the bargained wage equals the weighted sum of the average and the marginal product of labour, where the weights are given by the bargaining power of both parties.

Equilibrium wage and employment levels $\left(w^N, L^N \right)$ are determined by (8) and (9), i. e. by the intersection of the contract curve and the rent-division curve and thereby endogenising $u(\bar{w})$. These values moreover determine the equilibrium unemployment rate φ^N.

3.2. KS Solution

In the following, we analyse the labour market outcome under the assumption that bargaining follows the KS solution. Kalai & Smorodinsky (1975) suggest a solution where both parties make equal proportional concessions from their respective favoured points. Furthermore, this solution must be Pareto-efficient. The KS solution is determined by the intersection of the so-called KS curve and the Pareto curve.

The KS curve is implicitly defined as

$$(1-\delta)\frac{v_1 - \bar{v}_1}{v_1^* - \bar{v}_1} = \delta\frac{v_2 - \bar{v}_2}{v_2^* - \bar{v}_2}, \tag{10}$$

with v_i denoting the utility from bargaining for each party ($i = 1, 2$). The utopia point, i.e. the maximally attainable utility, is denoted by v_i^*, and the utility if bargaining breaks down by $\bar{v}_i$. In order to introduce different bargaining strengths for both parties, we follow the asymmetric axiomatic solution proposed by Dubra (2001). As in the Nash approach, δ and $1 - \delta$ denote union's and firm's bargaining power, respectively.

The Pareto curve defines all individually rational outcomes such that union's indifference curves and firm's isoprofit curves are tangent to each other. We obtain the Pareto curve by total differentiation of firm's profit function (1) and union's utility function (4). It is easy to see that the Pareto curve is equivalent to the contract curve (8) in the Nash solution.

In order to describe the labour market outcome, we apply the formal concept of the KS solution to the union bargaining model. Therefore, (10) has to be specified as follows. The utility from reaching a bargain for both parties is described by union's utility (4) and firm's profit function (1):

$$v_1 = lu(w) + (1-l)u(\bar{w})$$
$$v_2 = f(L) - wL,$$

where the subscripts 1 and 2 stand for the union and the firm, respectively. The respective utility in the case of disagreement is given by

$$\bar{v}_1 = u(\bar{w})$$
$$\bar{v}_2 = 0.$$

In a next step, we calculate both parties' utopia points. That is, we compute the respective optimal wage and employment levels the union and the firm want to achieve. First, a union's maximum feasible gain is determined by the highest wage the firm would be willing to pay without closing down. This can be shown by maximising union utility subject to the condition

that firm's profit is at least zero:

$$\max_{w,L} \; U = lu(w) + (1-l)u\left(\bar{w}\right) \tag{11}$$

$$\text{s.t.} \quad f(L) - wL = 0.$$

The maximising values for v_1^* can be obtained from the first-order conditions of (11). It is easy to see that the solution lies on the contract curve (8). Furthermore, since firms are left with zero profits, the solution equals the average product of labour. That is, the optimal wage and employment levels from the union's point of view (w^*, L^*) are implicitly given by the intersection of the contract and the average product of labour curve.

The utopia point of the firm v_2^* is determined by the minimum wage. Since the union's outside option is a weighted average of the minimum wage and unemployment benefit, the minimum wage is higher than the outside option. That is, the firm can at best reduce the wage to its statutory minimum $\tilde{w}$. This results in a smaller maximum feasible gain of the firm, which reduces its claim to the bargaining set. Moreover, because of the Pareto-efficiency of the bargaining outcome, the utopia point lies on the contract curve. The minimum wage level on the contract curve therefore determines the firm's optimal wage and employment levels $(\tilde{w}, \tilde{L})$. This result is in contrast to Gerber & Upmann (2006) where the firm's utopia point equals the union's outside option.

Substituting the values for v_i, $\bar{v}_i$ and v_i^* into (10), we finally end up with the asymmetric KS curve:

$$(1-\delta)\frac{l\left(u(w) - u\left(\bar{w}\right)\right)}{l^*\left(u\left(w^*\right) - u\left(\bar{w}\right)\right)} = \delta\frac{f(L) - wL}{f(\tilde{L}) - \tilde{w}\tilde{L}} \tag{12}$$

with $l^* \equiv \frac{L^*}{N}$. Hence, the bargaining outcome (w^{KS}, L^{KS}) is implicitly defined by (8) and (12). As in the Nash solution, the bargaining outcome determines the equilibrium unemployment rate φ^{KS}.

Figure 1 pictures possible wages and employment in the unionised sector under both, the Nash and the KS approach. Both solutions lie on the contract curve (CC). The Nash result (w^N, L^N) is described by the intersection with the rent-division curve (RDC). The intersection of the contract curve with the KS curve (KSC) characterises the outcome under the KS solution (w^{KS}, L^{KS}). Since outcomes below the minimum wage are not part of the bargaining set (dashed part of the contract curve), the KS solution lies between the utopia points of the firm $(\tilde{w}, \tilde{L})$ and the union (w^*, L^*).

A priori, it is not clear which bargaining approach generates a higher wage or employment level. In general, the solution depends on the the
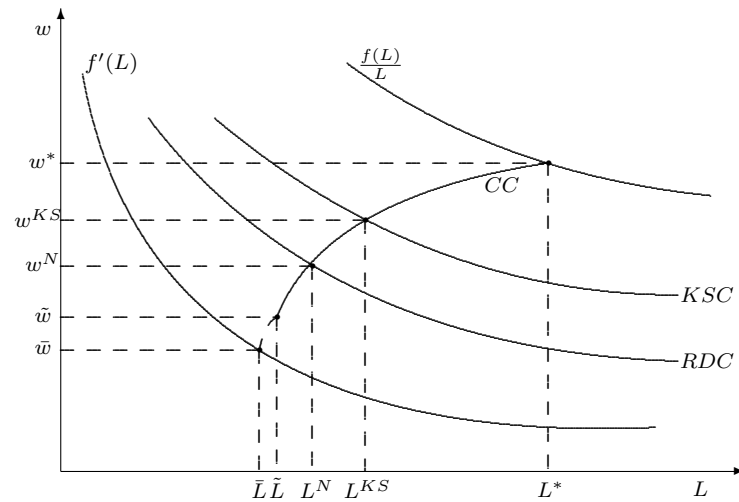
FIG. 1. Labour market outcomes of the Nash and the KS solution.

specific form of worker's utility and firm's production function. Both approaches, however, lead to the same result if the range between the respective utopia points and the outside options is the same for the union and the firm. Then the denominators in the KS curve (12) are equal on both sides. The relative gains of both parties weighted with the respective bargaining power change to weighted absolute gains. Furthermore, the outcome of the KS and the Nash solution might equal in that case only if workers are risk-neutral.[7] Comparing the rent-division curve (9) and the KS curve (12) shows that the degree of risk-aversion plays an important role in the KS solution while it does not in the Nash solution.

4. LABOUR MARKET EFFECTS OF RAISING THE MINIMUM WAGE AND UNEMPLOYMENT BENEFIT

The main focus of the paper is on discussing the labour market effect of increases in the minimum wage and unemployment benefit. Therefore, we analyse the changes in the union wage, employment in both sectors and the total unemployment rate due to changes in the minimum wage and unemployment benefit in both the Nash and the KS bargaining solution. Doing so it should be noted that although the alternative income is treated as data at the firm level, it is an endogenous variable at the macroeconomic level.

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[7]See Gerber & Upmann (2006) for a similar graphical illustration and an example with linear utility functions yielding the same result under both approaches.

The first and most obvious result is that a minimum wage increase causes less employment in the minimum wage sector. This can easily be shown by differentiating the marginal productivity condition in the second sector (2) with respect to the minimum wage:

$$M'(\tilde{w}) = g''^{-1}(\tilde{w}) < 0.$$

4.1. Nash Solution

The effects of increases in the minimum wage and the unemployment benefit under Nash bargaining can be calculated by differentiating the contract curve (8) and the rent-division curve (9). Rewriting these functions yields

$$\psi^{CC} = u(w) - u(\bar{w}) - u'(w)(w - f'(L)) = 0 \qquad (13)$$
$$\psi^{RDC} = \delta\frac{f(L)}{L} + (1-\delta)f'(L) - w \qquad\quad = 0 \qquad (14)$$

Using $u(\bar{w}) = pu(\tilde{w}) + (1-p)u(b)$, with $\tilde{w} > b$ and $p = \frac{M(\tilde{w})}{Z - \lambda L}$, the partial derivatives of (13) and (14) are as follows:

$$\psi_w^{CC} = -u''(w)(w - f'(L)) \qquad\qquad\quad > 0$$
$$\psi_w^{RDC} = -1 \qquad\qquad\qquad\qquad\qquad < 0$$
$$\psi_L^{CC} = -u'(\bar{w})\frac{\partial\bar{w}}{\partial L} + u'(w)f''(L) \qquad < 0$$
$$\psi_L^{RDC} = \frac{\delta}{L}\left(f'(L) - \frac{f(L)}{L}\right) + (1-\delta)f''(L) < 0$$
$$\psi_{\tilde{w}}^{CC} = -u'(\bar{w})\frac{\partial\bar{w}}{\partial\tilde{w}} \qquad\qquad\qquad \lesseqgtr 0$$
$$\psi_{\tilde{w}}^{RDC} \qquad\qquad\qquad\qquad\qquad\qquad = 0$$
$$\psi_b^{CC} = -(1-p)u'(b) \qquad\qquad\qquad < 0$$
$$\psi_b^{RDC} \qquad\qquad\qquad\qquad\qquad\qquad = 0$$

While the other derivatives have unambiguous signs, $\psi_{\tilde{w}}^{CC}$ needs some discussion. The ambiguous sign follows from the endogeneity of $p(\tilde{w}, L)$ at the macroeconomic level. Rewriting this expression in terms of elasticities gives

$$\psi_{\tilde{w}}^{CC} = -pu'(\tilde{w})\left(1 + \frac{\epsilon}{\eta}\right) \quad \begin{cases} > 0 & \text{if} \quad \epsilon + \eta < 0 \\ < 0 & \text{if} \quad \epsilon + \eta > 0, \end{cases} \qquad (15)$$

where $\epsilon \equiv \frac{\partial p}{\partial\tilde{w}}\frac{\tilde{w}}{p} = M'(\tilde{w})\frac{\tilde{w}}{M} < 0$ is the elasticity of labour demand in the second sector, and $\eta \equiv u'(\tilde{w})\frac{\tilde{w}}{u(\tilde{w}) - u(b)} > 0$ is the elasticity of the utility

gain from employment in the second sector with respect to the minimum wage, respectively. According to (15) $\psi_{\tilde{w}}^{CC}$ has a negative sign if $\epsilon + \eta > 0$. Concerning the elasticity of labour demand ϵ, Hamermesh (1993) surveys a large number of studies, where he shows that [-0.15, -0.75] is a reasonable confidence interval. Labour demand is thus rather inelastic in most labour markets. Furthermore, in the special case that there is no unemployment benefit and workers are risk neutral, we have $\eta = 1$. In a more general case, however, η will differ from unity. These impacts of the elasticity of labour demand and workers' risk attitude in a wage setting framework are discussed by Danziger (2009). He analyses under which assumptions low-wage workers benefit from an increase in their total wage income. In the following, we will show that the sign of these elasticities have important implications regarding the labour market effects under both bargaining solutions.

Summing up, the labour market effects of minimum wage and unemployment benefit increases are summarised in

PROPOSITION 1.   *(i) Under Nash bargaining over wage and employment, a minimum wage increase leads to a higher (lower) wage and to a lower (higher) employment level in the unionised sector if $\epsilon + \eta > (<) 0$.*
(ii) Higher unemployment benefits yield a higher wage and lower employment in the unionised sector.

Proof.   See Appendix.   ▋

The economic rationale behind the results presented in Proposition 1 follows from the fact that the minimum wage and unemployment benefit affect the outside option in different ways. Raising the unemployment benefit increases union's outside option and thus the bargained wage. Raising the minimum wage, however, can increase or decrease the outside option depending on the elasticities described above. The resulting changes in the unemployment rate are summarised in

PROPOSITION 2.   *(i) Under Nash bargaining, a higher minimum wage increases unemployment if $\epsilon + \eta > 0$. Otherwise, if $\epsilon + \eta < 0$, the effect is ambiguous.*
(ii) Higher unemployment benefit increases the unemployment rate.

Proof.   See Appendix.   ▋

Figures 2 and 3 picture these results indicating the labour market outcome in both sectors under Nash bargaining. In figure 2, a minimum wage increase from $\tilde{w}$ to $\tilde{w}'$ causes an employment reduction from M to M' in

the second sector. The workers' outside option rises (if $\epsilon + \eta > 0$) or falls (if $\epsilon + \eta < 0$). Accordingly, the contract curve shifts upwards (case ①) or downwards (case ②). Since the rent-division curve remains constant, the new wage and aggregate employment levels in the unionised sector are given by $w^{N'}$ and $\lambda L^{N'}$. In case ①, unemployment increases from Φ^N to $\Phi^{N'}$. In case ②, however, the possible positive employment effect in the unionised sector may dominate the negative employment effect in the second sector (therefore marked with ⑦).
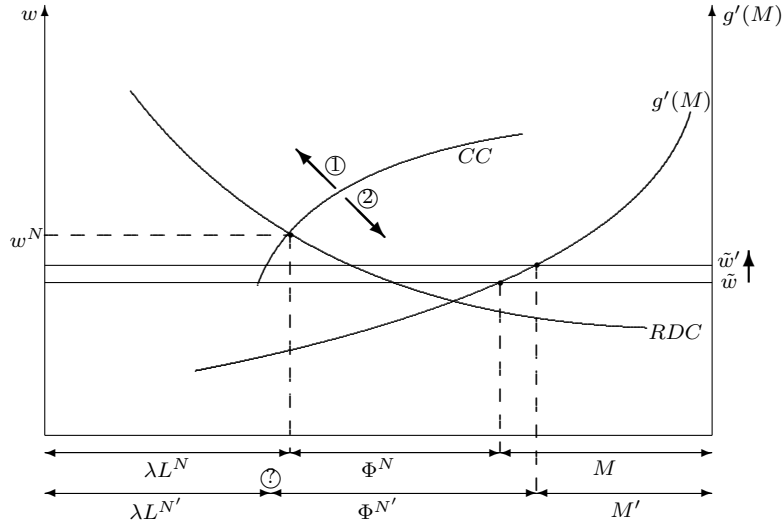

FIG. 2. Labour market effects of minimum wage increases in the Nash solution.

In figure 3, an increase in the unemployment benefit from b to b' has no effect on second sector employment, but shifts the contract curve upwards (case ③). Employment in the unionised sector decreases and thus aggregate unemployment increases.

4.2. KS Solution

The labour market effects of increases in the minimum wage and the unemployment benefit if bargaining follows the KS solution can be calculated by differentiating the contract curve (8) and the KS curve (12). Rearranging the KS curve (12) yields the following implicit function:

$$\psi^{KSC} = (1 - \delta)L\left[u(w) - u(\bar{w})\right]\left[f(\tilde{L}) - \tilde{w}\tilde{L}\right]$$
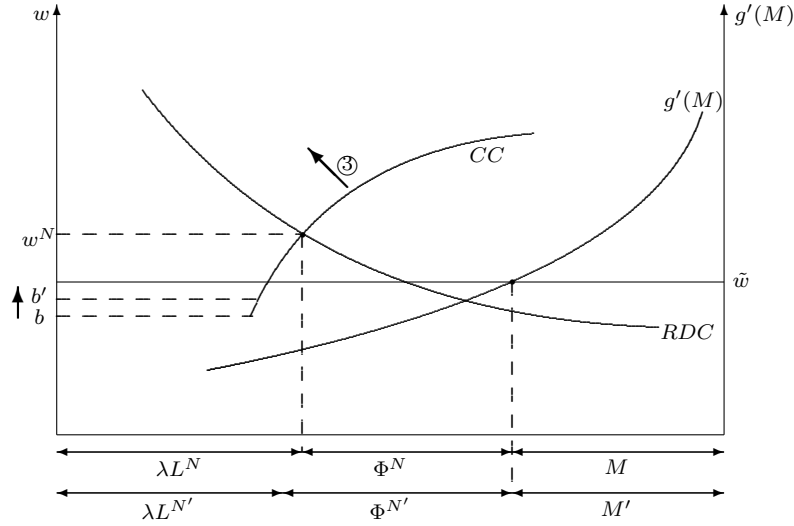$$- \delta L^*\left[u(w^*) - u(\bar{w})\right]\left[f(L) - wL\right] = 0. \tag{16}$$

FIG. 3. Labour market effects of unemployment benefit increases in the Nash solution.

Using the endogenous values for $u(\bar{w})$ and p and applying the envelope theorem, the partial derivatives are

$$\psi_w^{KSC} = (1-\delta)Lu'(w)\left[f(\tilde{L}) - \tilde{w}\tilde{L}\right] + \delta LL^*\left[u(w^*) - u(\bar{w})\right] > 0$$

$$\psi_L^{KSC} = (1-\delta)\left[u(w) - u(\bar{w})\right]\left[f(\tilde{L}) - \tilde{w}\tilde{L}\right]$$
$$+ \delta L^*\left[u(w^*) - u(\bar{w})\right]\left[w - f'(L)\right] - u'(\bar{w})\frac{\partial \bar{w}}{\partial L}X \qquad \lessgtr 0$$

$$\psi_{\tilde{w}}^{KSC} = -(1-\delta)L\tilde{L}\left[u(w) - u(\bar{w})\right] - u'(\bar{w})\frac{\partial \bar{w}}{\partial \tilde{w}}X \qquad \lessgtr 0$$

$$\psi_b^{KSC} = -(1-p)u'(b)X \qquad\qquad\qquad < 0$$

where $X \equiv (1-\delta)L\left[f(\tilde{L}) - \tilde{w}\tilde{L}\right] - \delta L^*\left[f(L) - wL\right] > 0$. The sign of X is straightforward if we rewrite the expression as $(1-\delta)\frac{L}{L^*} > \delta\frac{f(L)-wL}{f(\tilde{L})-\tilde{w}\tilde{L}}$. A comparison with the KS curve indicates that the inequality must hold since in (12) there is $u(w) - u(\bar{w}) < u(w^*) - u(\bar{w})$.

The ambiguous sign of ψ_L^{KSC} follows from the endogeneity of $u(\bar{w})$ in equilibrium. Since $u'(\bar{w})\frac{\partial \bar{w}}{\partial L} > 0$, the last term in ψ_L^{KSC} is negative while the first and second are positive. Hence, the KS curve may shift upwards or downwards depending on the appropriate employment level.

Furthermore, as discussed above, the ambiguous sign of $\psi_{\tilde{w}}^{KSC}$ depends on the elasticities ϵ and η. Using (15) we can rewrite this expression as

follows:

$$\psi_{\tilde{w}}^{KSC} = \psi_{\tilde{w}}^{CC} X - (1-\delta) L \tilde{L} \left[u(w) - u(\bar{w}) \right] \begin{cases} < 0 & \text{if} \quad \epsilon + \eta > 0 \\ < 0 & \text{if} \quad \epsilon + \eta < 0 \quad \text{and} \\ & \quad (1-\delta) L \tilde{L} \left[u(w) - u(\bar{w}) \right] > \psi_{\tilde{w}}^{CC} X \quad (17) \\ > 0 & \text{if} \quad \epsilon + \eta < 0 \quad \text{and} \\ & \quad (1-\delta) L \tilde{L} \left[u(w) - u(\bar{w}) \right] < \psi_{\tilde{w}}^{CC} X \end{cases}$$

To calculate the labour market implications of higher minimum wage and unemployment benefit, we need to know the sign of the Jacobian:

$$|J^{KS}| = \left| \begin{array}{cc} \psi_w^{CC} & \psi_L^{CC} \\ \psi_w^{KSC} & \psi_L^{KSC} \end{array} \right| \lessgtr 0.$$

This ambiguous result follows from $\psi_L^{KSC} \lessgtr 0$. If we have a closer look at the comparative statics, we can conclude that the effects are not clear if $\psi_L^{KSC} < 0$. For this case, we cannot derive explicit differences between Nash and KS since there are too many ambiguous effects. However, if $\psi_L^{KSC} > 0$, some more distinct results can be derived. In order to highlight the comparative static differences between the two bargaining solutions, we restrict our analysis to the case where $\psi_L^{KSC} > 0$. Then the Jacobian has a positive sign and the wage effects can be summarised in

PROPOSITION 3. *(i) Under KS bargaining over wage and employment, a minimum wage increase leads to a higher wage in the unionised sector if $\epsilon + \eta > 0$.*
(ii) If $\epsilon + \eta < 0$, a higher minimum wage leads to a lower wage if moreover $\psi_{\tilde{w}}^{KSC} > 0$. If $\epsilon + \eta < 0$ and $\psi_{\tilde{w}}^{KSC} < 0$, the wage effect is ambiguous.
(iii) Higher unemployment benefits yield a higher wage in the unionised sector.

Proof.   See Appendix. ▌

The employment effects in the unionised sector are summarised in

PROPOSITION 4. *(i) Under KS bargaining over wage and employment, a minimum wage increase leads to more employment in the unionised sector if $\epsilon + \eta < 0$ and $\psi_{\tilde{w}}^{KSC} < 0$. Otherwise, the employment effects are ambiguous.*
(ii) Higher unemployment benefits yield more or less employment in the unionised sector.

Proof.   See Appendix. ▌

Summing up, in the Nash approach a minimum wage increase neither changes the marginal nor the average product of labour. The distribution

given by the rent-division curve remains constant. In the KS solution, however, the outside options and the utopia points affect the distribution of rents represented by the KS curve. The difference between these two labour market policies comes from the effect on the bargaining parameters, respectively. Higher unemployment benefit increases union's outside option and shifts the KS curve upwards. This result replicates the similar result in Gerber & Upmann (2006). A minimum wage increase, however, affects union's outside option whereas the sign of this effect is not clear. Moreover, firm's utopia point is reduced by a higher minimum wage. Therefore, the KS curve shifts upwards or downwards depending on union's utility and firm's production functions.

A consolidated view indicates that the unemployment consequences of raising the unemployment benefit and the minimum wage are ambiguous if bargaining follows the KS solution. This result follows directly from differentiating equation (3) and Proposition 4. Employment in the second sector declines due to the higher minimum wage and keeps constant if the unemployment benefit increases.[8] Employment in the first sector may rise if the shift of the KS curve is large enough to countervail the upward shift of the contract curve. If this positive employment effect is larger than the negative effect in the second sector, overall unemployment declines.

Figures 4 and 5 illustrate these results. In figure 4, a higher minimum wage affects not only the contract curve but also the KS curve because the union's outside option and the firm's utopia point change. As discussed above, this shift may be upwards or downwards (①' and ②').

In figure 5, an increase in the unemployment benefit from b to b' shifts both the contract curve (case ③) and the KS curve (case ③') upwards. The bargained wage increases but the employment effect in the unionised sector is ambiguous (marked with ②).

5. CONCLUSION

The paper addresses the effects of two labour market policies–a statutory minimum wage and unemployment benefit–in a dual labour market. Though the minimum wage is only binding in the second sector, it also affects the unionised sector by serving as part of the bargaining parameters. The crucial feature of the model is the different impact of a minimum wage increase and an increase in the unemployment benefit under two different bargaining approaches, the Nash and the KS solution. We point out quantitative wage and employment differences under both solutions. However, more important with regard to policy implications are the different

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[8]Of course there will be some additional effects if unemployment is financed by labour taxes in general equilibrium. We ignore these effects here for reasons of simplicity.
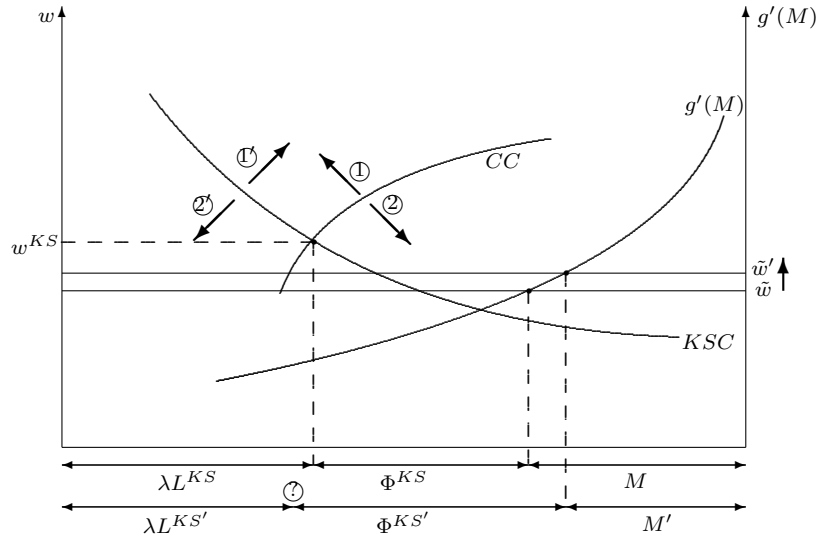
FIG. 4. Labour market effects of minimum wage increases in the KS solution.
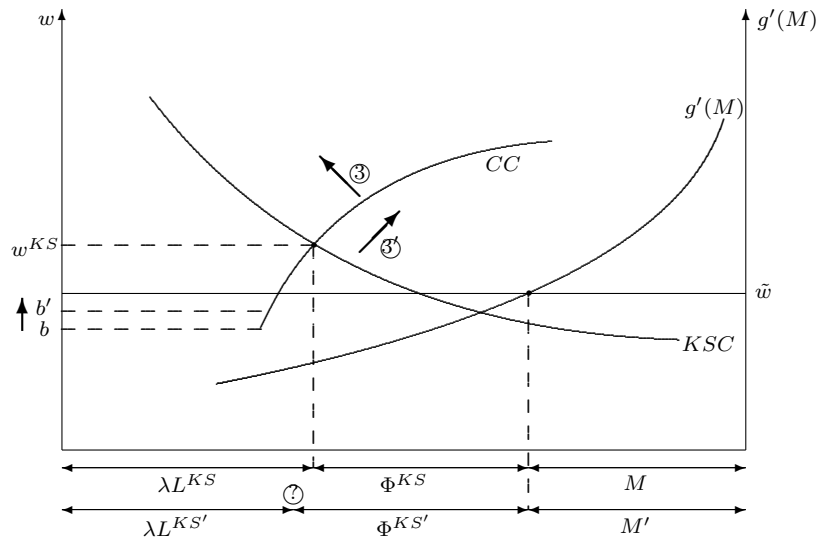

FIG. 5. Labour market effects of unemployment benefit increases in the KS solution.

qualitative effects. Under Nash bargaining, higher unemployment benefit leads to a higher union wage and to a lower employment level. The wage and employment effects of a higher minimum wage, however, depend on

the elasticity of labour demand in the second sector and the elasticity of the utility gain from employment in the second sector with respect to the minimum wage, respectively.

Under KS bargaining, the wage and employment effects are more ambiguous. However, we can show that there are some additional effects affecting the distribution of rents in this bargaining solution. First, raising the unemployment benefit and raising the minimum wage affect the outside option and therefore shift the KS curve. This shift can be upwards or downwards depending on union's utility and firm's production functions. Second, a higher minimum wage reduces firm's utopia point therefore also affecting the distribution of rents. Hence, policy implications on changing the unemployment benefit or the minimum wage depend on whether negotiations between unions and firms follow the Nash solution or the KS solution.

APPENDIX A

Proof of Proposition 1.

From the first order conditions it turns out that the Jacobian has a negative sign:

$$|J^N| = \left| \begin{array}{cc} \psi_w^{CC} & \psi_L^{CC} \\ \psi_w^{RDC} & \psi_L^{RDC} \end{array} \right| < 0.$$

The wage and employment effects in the unionised sector due to a minimum wage increase are obtained by using Cramer's rule:

$$\frac{dw^N}{d\tilde{w}} = \frac{\left| \begin{array}{cc} -\psi_{\tilde{w}}^{CC} & \psi_L^{CC} \\ -\psi_{\tilde{w}}^{RDC} & \psi_L^{RDC} \end{array} \right|}{|J^N|} \quad \begin{cases} > 0 & \text{if} \quad \epsilon + \eta > 0 \\ < 0 & \text{if} \quad \epsilon + \eta < 0 \end{cases}$$

$$\frac{dL^N}{d\tilde{w}} = \frac{\left| \begin{array}{cc} \psi_w^{CC} & -\psi_{\tilde{w}}^{CC} \\ \psi_w^{RDC} & -\psi_{\tilde{w}}^{RDC} \end{array} \right|}{|J^N|} \quad \begin{cases} < 0 & \text{if} \quad \epsilon + \eta > 0 \\ > 0 & \text{if} \quad \epsilon + \eta < 0 \end{cases}$$

$$\frac{dw^N}{db} = \frac{\left| \begin{array}{cc} -\psi_b^{CC} & \psi_L^{CC} \\ -\psi_b^{RDC} & \psi_L^{RDC} \end{array} \right|}{|J^N|} \quad > 0$$

$$\frac{dL^N}{db} = \frac{\left| \begin{array}{cc} \psi_w^{CC} & -\psi_b^{CC} \\ \psi_w^{RDC} & -\psi_b^{RDC} \end{array} \right|}{|J^N|} \quad < 0. \quad \blacksquare$$

Proof of Proposition 2.

These results follow directly from differentiating equation (3) and Proposition 1. The change in the unemployment rate with respect to changes in the minimum wage and unemployment benefit are given by

$$\frac{d\varphi^N}{d\tilde{w}} = -\frac{\lambda\frac{dL^N}{d\tilde{w}} + \frac{dM}{d\tilde{w}}}{Z} \quad \begin{cases} > 0 & \text{if } \epsilon + \eta > 0 \\ > 0 & \text{if } \epsilon + \eta < 0 \text{ and } -\lambda\frac{dL^N}{d\tilde{w}} > \frac{dM}{d\tilde{w}} \\ < 0 & \text{if } \epsilon + \eta < 0 \text{ and } -\lambda\frac{dL^N}{d\tilde{w}} < \frac{dM}{d\tilde{w}} \end{cases}$$

$$\frac{d\varphi^N}{db} = -\frac{\lambda\frac{dL^N}{db}}{Z} \qquad > 0. \quad \blacksquare$$

Proof of Proposition 3.

Using Cramer's rule and assuming $\psi_L^{KSC} > 0$ yields

$$\frac{dw^{KS}}{d\tilde{w}} = \frac{\begin{vmatrix} -\psi_{\tilde{w}}^{CC} & \psi_L^{CC} \\ -\psi_{\tilde{w}}^{KSC} & \psi_L^{KSC} \end{vmatrix}}{|J^{KS}|} \quad \begin{cases} > 0 & \text{if } \epsilon + \eta > 0 \\ \lessgtr 0 & \text{if } \epsilon + \eta < 0 \text{ and } \psi_{\tilde{w}}^{KSC} < 0 \\ < 0 & \text{if } \epsilon + \eta < 0 \text{ and } \psi_{\tilde{w}}^{KSC} > 0 \end{cases}$$

$$\frac{dw^{KS}}{db} = \frac{\begin{vmatrix} -\psi_b^{CC} & \psi_L^{CC} \\ -\psi_b^{KSC} & \psi_L^{KSC} \end{vmatrix}}{|J^{KS}|} \qquad > 0. \quad \blacksquare$$

Proof of Proposition 4.

Using Cramer's rule yields

$$\frac{dL^{KS}}{d\tilde{w}} = \frac{\begin{vmatrix} -\psi_w^{CC} & -\psi_{\tilde{w}}^{CC} \\ -\psi_w^{KSC} & -\psi_{\tilde{w}}^{KSC} \end{vmatrix}}{|J^{KS}|} \quad \begin{cases} > 0 & \text{if } \epsilon + \eta < 0 \text{ and } \psi_{\tilde{w}}^{KSC} < 0 \\ \lessgtr 0 & \text{if } \epsilon + \eta < 0 \text{ and } \psi_{\tilde{w}}^{KSC} > 0 \\ \lessgtr 0 & \text{if } \epsilon + \eta > 0 \end{cases}$$

$$\frac{dL^{KS}}{db} = \frac{\begin{vmatrix} -\psi_w^{CC} & \psi_b^{CC} \\ -\psi_w^{KSC} & \psi_b^{KSC} \end{vmatrix}}{|J^{KS}|} \qquad \lessgtr 0. \quad \blacksquare$$

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