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Robust Consumption and Portfolio Choice with Habit Formation, the Spirit of Capitalism and Recursive Utility*

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This paper studies consumption and portfolio choice with habit formation, the spirit of capitalism, recursive utility and robustness in a continuous-time stochastic model and examines how the four factors affect consumption and portfolio choice, consumption dynamics and asset pricing. The explicit solutions of the robust consumption and portfolio choice problem are obtained, the implications of the four factors for consumption and portfolio choice are discussed, and then the dynamics of consumption and the formula of asset pricing are derived. It is shown that the combined effects of habit formation which stems from past consumption, the spirit of capitalism which endows investors with direct wealth preferences, recursive utility which allows the separation of risk aversion and intertemporal substitution and robustness which takes account of model uncertainty can better interpret the consumption smoothing puzzle and the equity premium puzzle than cases which only consider one factor.

Key Words: Consumption and portfolio choice; Habit formation; The spirit of capitalism; Recursive utility; Robustness.
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1. INTRODUCTION

Consumption and portfolio choice is a classical problem of financial economics. In two pioneering papers, Merton (1969, 1971) introduced stochastic control techniques to analyze consumption and portfolio choice in continuous-time models. Hereafter there has been an increased interest among both academics and practitioners in finding optimal consumption and portfolio strategies, such as Schroder and Skiadas (1999, 2003, 2005), Chacko and Viceira (2005), Liu (2007), Bekaert et al. (2009) and Liu (2010, 2011, 2013). There are several remarkable aspects to be noticed in the literature.

Firstly, the last few years we have seen a renewal of interest in the old idea that habit may play a key role in consumption and portfolio choice. The idea underlying this literature is that through the process of habit formation, one's past consumption might influence the utility yielded by current consumption, see Carroll et al. (2000). In the habit formation literature, Sundaresan (1989), Constantinides (1990), Campbell and Cochrane (1999), Masten (2003) and Munk (2008) analyze the implications of habit formation on consumption and portfolio choice.

Secondly, the spirit of capitalism which has been characterized as capitalists accumulate wealth for the sake of wealth by Weber (1958) and Keynes (1971) has been used to address consumption and portfolio choice. Bakshi and Chen (1996) has explored empirically the relationship between the spirit of capitalism and stock market pricing and offered an attempt towards the resolution of the equity premium puzzle in Mehra and Prescott (1985). They have shown that when investors care about status they will be more conservative in risk taking and more frugal in consumption spending. Furthermore, stock prices tend to be more volatile with the presence of the spirit of capitalism. The extensions include Yang (1999), Smith (2001), Gong and Zou (2002), Kenc and Dibooğlu (2007) and Roussanov (2010).

Thirdly, recursive utility has been used to interpret the equity premium puzzle by disentangling risk aversion and intertemporal substitution. For the analysis convenience, most related papers assume that the investor's utility is represented by an additively time-separable expected utility function which the intertemporal elasticity of substitution and the coefficient of relative risk aversion are constrained to be reciprocals of one another. These highly unrealistic preferences of the investor easily lead to a misapprehension of the role of preference parameters, especially misunderstanding the risk aversion degree of investor. Svensson (1989) and Weil (1989) first investigate consumption and portfolio choice for an investor with recursive utility. After Obstfeld (1994) and Smith (1996), recursive utility often appears in the consumption and portfolio choice theory instead of expected utility, see Schroder and Skiadas (2003, 2005), Bhamra and Uppal (2006), Kraft et al. (2011) and Buss et al. (2011).

Finally, the bulk of the literature on consumption and portfolio choice assumes that investors have complete confidence in the probability law governing the evolution of state variables and do not worry about model uncertainty. Recently, a growing literature begins to concern about the implications of model uncertainty for consumption and portfolio choice. Maenhout (2004) employs the robust control approach of Anderson et al. (2003) to examine consumption and portfolio choice under model uncertainty and seeks robust decisions. Maenhout (2006) and Liu (2010) analyze the optimal intertemporal portfolio choice of an investor who worries about model misspecification and insists on robust decision rules when facing a mean-reverting risk premium for the constant relative risk aversion (CRRA) utility and stochastic differential utility respectively. Miao (2009) studies optimal consumption and portfolio choice in a Merton-style model with incomplete information when there is a distinction between model uncertainty and risk by adoption by recursive multiple-prior utility. Liu (2011) examines consumption and portfolio choice under model uncertainty, where expected returns of a risky asset follow an unobservable hidden Markov chain.

In view of the facts that almost all related papers only consider one factor among habit formation, the spirit of capitalism, recursive utility and robustness (or model uncertainty), we study consumption and portfolio choice together with these four factors in a continuous-time model and analyze consumption dynamics and asset pricing. Our paper is distinguished from many other papers on consumption and portfolio choice by five features. Firstly, we choose the habit formation expression in Masten (2003) other than reference Constantinides (1990) in the model of consumption and portfolio choice. With habit persistence, the optimal portfolio between risky and riskless securities does not remain constant as the investor ages. Instead, optimal portfolio allocation varies with habit and wealth. Secondly, in view of the prosperous literature of the spirit of capitalism which acknowledges that people are concerned with their standing in society, we introduce the spirit of capitalism into our model and analyze how it affects the investor's consumption and asset pricing. When investors care about social status and the associated risks of falling out of social status, they will hedge against these risks. This can have potentially important consumption and portfolio allocation effects. Thirdly, our model extends along the literature of recursive utility to include a recursive utility function that disentangles risk aversion from intertemporal substitution-thereby enabling an analysis of the distinct roles played by investors' attitudes towards risk (the desire to smooth consumption across states of nature) and intertemporal substitution (the desire to smooth consumption across time). Fourthly, we extend our model to a model uncertainty case, in which the investor worries about model misspecification and considers some endogenous worst-case model among a family of alternative models surrounding the reference model. In accordance with max-min utility, the investor seeks robust decision rules along the lines of Anderson et al. (2003) and Maenhout (2004, 2006). Fifthly, we have an exact continuous-time stochastic model rather than the discrete-time stochastic approximations (through Markov chains) more commonly adopted in the related literature. Although continuous-time techniques are more restrictive compared to discrete-time techniques, they are largely favored in the consumption and portfolio choice literature because the results are more transparent and typically more insightful than those found from discrete-time analysis.

The rest of the paper is organized as follows. In section 2 we present a continuous-time parameterized-preference model with habit formation, the spirit of capitalism, recursive utility and robustness. In section 3 we describe the robust investor's consumption and portfolio choice problem and derive the explicit solutions to this problem. Section 4 derives consumption dynamics and analyzes the consumption smoothing puzzle. Section 5 briefly explores the implications of our framework for habit formation, the spirit of capitalism, recursive utility and robustness on asset pricing and tries to interpret the equity premium puzzle. Section 6 offers concluding remarks.

2. THE MODEL

In this section, we first present a basic consumption and portfolio choice model with habit formation, the spirit of capitalism and recursive utility. Then, we introduce robustness into the basic model and characterize the robust consumption and portfolio choice problem.

2.1. The Basic Model

We consider an economy with a continuum of identical, competitive infinitely-lived investors with total measure 1. All investors consume a single good.

2.1.1. Asset Returns and Wealth Constraint

Each investor faces portfolio choice of investing in a riskless asset which offers a sure instantaneous and constant yield r, or investing in a risky asset. The price of the risky asset $P(t)$, is governed by the following geometric Brownian motion

$$\frac{dP(t)}{P(t)} = \mu dt + \sigma dB(t), \tag{1}$$

where μ and σ are the conditional expectation value and standard deviation of the return rate on risky asset per unit time respectively, and $B(t)$ is a standard Brownian motion.

Let $W(t)$ denote the representative investor's real wealth, and $\theta(t)$ denote the fraction of wealth invested in the risky asset at time t, then the representative investor's wealth are subject to the following constraint

$$dW(t) = ([\theta(t)(\mu - r) + r]W(t) - c(t))dt + \theta(t)\sigma W(t)dB(t), \tag{2}$$

where $c(t)$ is the investor's private consumption flow.

2.1.2. *Habit Formation*

The investor cares about consumption relative to a "habit stock" determined by his past private consumption, and takes into account the effect of current consumption on the future habit stock. Following Sundaresan (1989), Masten (2003) and Munk (2008), the habit stock z_t is the exponentially declining weighted average of the past consumption rates, i.e.

$$z(t) = z_0 e^{-\beta t} + \beta \int_0^t e^{\beta(s-t)} c(s)ds \geq 0, \quad \beta \geq 0, \tag{3}$$

where $z_0 \geq 0$ is the initial habit stock. The parameter β determines the relative weights of consumption at different times. The larger is β, the more important is consumption in the recent past. Differentiating (3) implies the following relationship for the habit stock

$$dz(t) = \beta[c(t) - z(t)]dt. \tag{4}$$

2.1.3. *The Spirit of Capitalism and Recursive Utility*

The investor's utility is dependent on not only the current consumption rate $c(t)$, but also on the habit stock $z(t)$. In addition, the investor cares about his social status which is measured by his absolute wealth $W(t)$. This social status preference is called the spirit of capitalism, see Weber (1958) and Zou (1994, 1995). Inspired by Sundaresan (1989), Constantinides (1990), Masten (2003), Munk (2008) and Bakshi and Chen (1996), we specify the argument in the utility function as the algebra sum between consumption, habit and wealth, in that $c(t) - z(t) + \lambda W(t)$, where the parameter $\lambda \geq 0$ measures the strength of the spirit of capitalism. If $\lambda = 0$, there is no the spirit of capitalism and we recover the familiar case of surplus consumption $c(t) - z(t)$. In our setting the consumption rate is required to exceed the difference $z-\lambda W$ of the habit level and the preference wealth so that the difference $z-\lambda W$ plays the role of a minimum or subsistence consumption rate determined by past consumption rates and current wealth, and the constraint $c \geq z-\lambda W$ will be binding whenever the investor makes his decisions. In our model the consumption rate can be lower than the habit level ($c < z$), this is the remarkable difference with the traditional habit formation literature. Following Svensson (1989) and Obstfeld (1994), we consider the intertemporal objective function $U(c(t), z(t), W(t))$ defined by the recursive relation

$$\begin{aligned} & f([1-R]U(c_t, z_t, W_t)) \qquad (5) \\ = & \lim_{dt\to 0^+} \left\{ \frac{1-R}{1-1/\epsilon}(c_t - z_t + \lambda w_t)^{1-1/\epsilon} dt + e^{-\rho dt} f([1-R]E_t U(c_{t+dt}, z_{t+dt}, W_{t+dt})) \right\}, \end{aligned}$$

where $c_{t+dt} = c(t+dt), z_{t+dt} = z(t+dt), W_{t+dt} = W(t+dt)$ and the function $f(x)$ is given by

$$f(x) = \frac{1-R}{1-1/\epsilon} x^{\frac{1-1/\epsilon}{1-R}}.$$

In (5), E_t is a mathematical expectation conditional on time-t information, $\rho > 0$ is the rate of time preference, $R > 1$ is the coefficient of relative risk aversion, and $0 < \epsilon < 1$ is the elasticity of intertemporal substitution. When $R = 1/\epsilon$, then $f(x) = x$ and

$$U(c(t), z(t), W(t)) = E_t \int_t^\infty \frac{(c(t) - z(t) + \lambda W(t))^{1-R}}{1-R} e^{-\rho(s-t)} ds. \qquad (6)$$

This is the standard constant relative risk aversion (CRRA) utility, which does not allow independent variation in risk aversion and consumption substitutability over time. If there is no the spirit of capitalism, i.e. $\lambda = 0$, utility function (6) turns to be the power form as in Munk (2008).

2.2. Robustness

Along the line of Anderson et al. (2003) and Maenhout (2004, 2006), we view the basic model in Section 2.1 as the reference model. The investor accepts the reference model as useful, but suspects it to be misspecified and considers alternative models. The preference for robustness is achieved by having the investor guard against an adverse alternative model that is reasonably similar to the reference model.

The investor has an alternative wealth dynamics and no alternative habit formation because habit formation is locally deterministic. The distort law of wealth dynamics implied by an alternative model is given by

$$\begin{aligned} & dW(t) \qquad (7) \\ = \ & ([\theta(t)(\mu - r) + r]W(t) - c(t))dt + \theta(t)\sigma W(t)[\theta(t)\sigma W(t)u(W(t), z(t))dt + dB(t)] \\ = \ & ([\theta(t)(\mu - r) + r]W(t) - c(t) + \theta(t)^2\sigma^2 W(t)^2 u(W(t), z(t)))dt + \theta(t)\sigma W(t)dB(t), \end{aligned}$$

where $u(W(t), z(t))$ is an endogenous drift adjustment term to be determined from solving a robust optimization problem presented below.

To penalize the distance of an alternative model to the reference model, a penalty term is incurred in the investor's utility. For recursive utility we adopt the following distorted utility form

$$\begin{aligned} & f([1 - R]U^*(c_t, z_t, W_t)) \\ = \ & \lim_{dt \to 0^+} \left\{ (1 - R) \left[\frac{(c_t - z_t + \lambda W_t)^{1-1/\epsilon}}{1 - 1/\epsilon} + \frac{1}{2\Psi(W_t, z_t)} \theta_t^2 \sigma^2 W_t^2 u(W_t, z_t)^2 \right] dt \right. \\ + \ & \left. e^{-\rho dt} f([1 - R]E_t U^*(c_{t+dt}, z_{t+dt}, W_{t+dt})) \right\}, \qquad (8) \end{aligned}$$

where $U^*(c_t, z_t, W_t)$ is the robust intertemporal objective function, $\Psi(W_t, z_t) \geq 0$ is the robustness preference parameter and $\frac{1}{2\Psi(W_t, z_t)}\theta^2\sigma^2 W_t^2 u(W_t, z_t)^2 \geq 0$ is the relative entropy which measures the distance of the alternative model to the reference model. When $R = 1/\epsilon$, we have the CRRA utility form

$$\begin{aligned} & U^*(c(t), z(t), W(t)) \qquad (9) \\ = \ & E_t \int_t^\infty \left[\frac{(c_t - z_t + \lambda W_t)^{1-R}}{1 - R} + \frac{1}{2\Psi(W_t, z_t)} \theta_t^2 \sigma^2 W_t^2 u(W_t, z_t)^2 \right] e^{-\rho(s-t)} ds. \end{aligned}$$

This is similar to Maenhout (2004) without habit formation and the spirit of capitalism.

The robust investor worries about that an alternative model has an adverse effect on the non-expected utility and wants to consider the worst-case alternative model in making decision. Then the robust investor solves the following max-min optimization problem

$$\begin{cases} \max\limits_{c,\theta} \min\limits_{u} U^*(c(t), z(t), W(t)) \\ \text{subject to constraints (7) and (4).} \end{cases} \qquad (10)$$

3. ROBUST CONSUMPTION AND PORTFOLIO CHOICE

Let $V(W(t), z(t))$ denote the maximum feasible level of lifetime utility $U^*(c(t), z(t), W(t))$ starting from time t, the Hamilton-Jacobi-Bellman

(HJB) equation for the robust investor's stochastic optimization problem is given by the following equation

$$
\begin{aligned}
0 &= \max_{c,\theta} \min_{u} \left\{ (1-R) \left[\frac{(c-z+\lambda W)^{1-1/\epsilon}}{1-1/\epsilon} + \frac{1}{2\Psi(W,z)} \theta^2 \sigma^2 W^2 u(W,z)^2 \right] \right. \\
&- \left. \rho f([1-R]V) + (1-R) f'([1-R]V) \left(\frac{EdV}{dt} \right) \right\}, \qquad (11)
\end{aligned}
$$

where

$$
\frac{EdV}{dt} = ([\theta(\mu-r)+r]W - c + \theta^2\sigma^2 W^2 u(W,z))V_W + \beta(c-z)V_z + \frac{1}{2}\theta^2\sigma^2 W^2 V_{WW}, \qquad (12)
$$

V_W, V_z and V_{WW} denote the first and second differentials of $V(W,z)$ with respect to W and z.

Solving first for the minimization part of the optimal problem yields to

$$
u^*(W,z) = -\Psi(W,z) f'([1-R]V) V_W. \qquad (13)
$$

The robustness preference parameter $\Psi(W,z) \geq 0$ measures the strength of preference for robustness or the degree of confidence in the reference model. When $\Psi(W,z) = 0$, then $u^*(W,z) = 0$, that is, the investor desires no robustness or has complete faith in the reference model.

Substituting for $u^*(W,z)$ in the HJB equation (11) and (12) leads to the first-order optimality conditions for consumption and portfolio choice are given by

$$
\begin{aligned}
c &= z - \lambda W + [f'([1-R]V)(V_W - \beta V_z)]^{-\epsilon}, \qquad (14) \\
\theta &= \frac{\mu - r}{\sigma^2} \frac{V_W}{-W[V_{WW} - \Psi(W,z) f'([1-R]V) V_W^2]}. \qquad (15)
\end{aligned}
$$

Robustness does not affect the form of the optimality condition for consumption but do the portfolio rule.

In order to obtain an explicit solution, we assume the robustness preference parameter $\Psi(W,z)$ is state-dependent and has the following form

$$
\Psi(W,z) = \frac{\kappa}{(1-R) f'([1-R]V) V} \geq 0, \qquad (16)
$$

where the parameter $\kappa \geq 0$ denotes the preference for robustness or uncertainty aversion. Different with Maenhout (2004, 2006) and Liu (2010), $\Psi(W,z)$ is not scaled by the reciprocal of the value function $\frac{1}{V(W,z)}$ in our model.

THEOREM 1. *Given the admissible conditions:* $\lambda < h < 1/k_1$, $W_t - k_1 z_t > 0$ *and the assumption for* $\Psi(W, z)$ *in (16), the explicit solutions to the robust investor's optimization problem are characterized as*

$$\begin{aligned} u^*(W(t), z(t)) &= -\frac{\kappa}{W(t) - k_1 z(t)}, && (17) \\ c^*(t) &= z(t) - \lambda W(t) + h(W(t) - k_1 z(t)) \\ &= (h - \lambda)W(t) + (1 - hk_1)z(t), && (18) \\ \theta^*(t) &= \frac{\mu - r}{(R + \kappa)\sigma^2}\left[1 - \frac{k_1 z(t)}{W(t)}\right], && (19) \\ V(W(t), z(t)) &= \frac{k_0^{\frac{1-R}{1-\epsilon}}(W(t) - k_1 z(t))^{1-R}}{1 - R}, && (20) \end{aligned}$$

where

$$\begin{aligned} h &= \frac{1}{1 + \beta k_1}\left\{\epsilon\rho + (1 - \epsilon)\left[r + (1 + \beta k_1)\lambda + \frac{(\mu - r)^2}{2(R + \kappa)\sigma^2}\right]\right\}, \\ k_0 &= \frac{1}{(1 + \beta k_1)^{1-\epsilon}}\left\{\epsilon\rho + (1 - \epsilon)\left[r + (1 + \beta k_1)\lambda + \frac{(\mu - r)^2}{2(R + \kappa)\sigma^2}\right]\right\}, \\ k_1 &= \frac{1}{2\beta\lambda}[\sqrt{(r + \lambda)^2 + 4\beta\lambda} - (r + \lambda)]. \end{aligned}$$

Proof. See Appendix A. ▮

Robustness amounts to an increase in risk aversion. The nominal risk aversion R is replaced by $R + \kappa$. The optimal consumption strategy in (18) is to consume the current minimum level $(z - \lambda W)$ plus a fraction of the "free wealth" $(W - k_1 z)$ which is in excess of the costs of financing the future minimum consumption stream. The optimal portfolio weight θ^* in (19) is the risky investment in the tangency portfolio, which is represented by the Sharpe ratio $\frac{\mu - r}{\sigma}$. The investor-specific position in the tangency portfolio is determined by the relative risk tolerance $\frac{1}{R+\kappa}\left[1 - \frac{k_1 z}{W}\right]$. This property is accordance with Merton (1971), Constantinides (1990) and Bakshi and Chen (1996). These models are the special cases of our model. The investor-specific position in the tangency portfolio is the function of z/W. We can interpret z/W as the investor's relative habit level and k_1 as the importance of the reference set by past consumption. We find that

$$\frac{\partial k_1}{\partial \beta} = -\frac{k_1^2 \lambda}{2\beta k_1 \lambda + r + \lambda} < 0, \quad \frac{\partial k_1}{\partial \lambda} = -\frac{k_1(1 + \beta k_1)}{2\beta k_1 \lambda + r + \lambda} < 0.$$

The more important the recent past consumption and the spirit of capitalism, the lower important habit formation. This is not consistent with Sundaresan (1989), Masten (2003) and Munk (2008). Without the spirit of capitalism ($\lambda = 0$), we have $k_1 = 1/r$ and habit formation parameter β has no effect on k_1.

4. CONSUMPTION DYNAMICS AND THE CONSUMPTION SMOOTHING PUZZLE

In this section, we state the dynamics of consumption and investigate the consumption smoothing puzzle. The effective relative risk aversion (RRA) coefficient in the next section is a function of the ratio $z(t)/c(t)$. Theorem 2 states this ratio has a stationary distribution under some conditions and presents this distribution. This distribution will be used to calculate the unconditional mean of the effective RRA coefficient in the next section.

THEOREM 2. *Given the admissible assumption* $M = \frac{2(n+\beta k_1 \lambda)}{m^2\sigma^2} > 1$, *the equilibrium consumption growth rate is given by*

$$\frac{dc(t)}{c(t)} = \{(\beta+n)[1-(1-k_1\lambda)\xi(t)] - \beta k_1 \lambda\}dt + [1-(1-k_1\lambda)\xi(t)]m\sigma dB(t), \tag{21}$$

where k_1, h *are given by Theorem 1,* $\xi(t) = z(t)/c(t)$, *and*

$$m = \frac{\mu - r}{(R+\kappa)\sigma^2}, \quad n = \frac{R}{R+\kappa}m(\mu - r) + r - (1+\beta k_1)(h-\lambda), \tag{22}$$

$\xi(t)$ *has a stationary distribution with density*

$$\Psi(x) = \frac{N^{M_1}}{\Gamma(M-1)}x^{-2}\left[\frac{1-(1-k_1\lambda)x}{x}\right]^{M-2}\exp\left\{-N\left[\frac{1-(1-k_1\lambda)x}{x}\right]\right\},$$
$$0 \le x \le \frac{1}{1-k_1\lambda},$$

where $N = \frac{2\beta}{m^2\sigma^2}$.

Proof. See Appendix B. ▌

To understand habit formation, the spirit of capitalism, recursive utility and robustness deeply, we use the method of Sundaresan (1989) to interpret the consumption smoothing puzzle. We need to prove that the consumption volatility is lower than the wealth volatility. The ratio of the standard derivation of consumption growth and wealth growth, η, is given by

$$\eta = \frac{\mathrm{std}(dc)}{\mathrm{std}(dW)} = \frac{1-(1-k_1\lambda)\xi}{1-k_1 z/W}\frac{c}{W} = h - \lambda.$$

The effects of habit formation on the ratio of the standard derivation of consumption growth and wealth growth is characterized as follows

$$\frac{\partial \eta}{\partial \beta} = -\frac{\epsilon\rho + (1-\epsilon)\left[r + \beta k_1\lambda + \frac{(\mu-r)^2}{2(R+\kappa)\sigma^2}\right]}{(1+\beta k_1)^2(2\beta k_1\lambda + r + \lambda)} < 0.$$

An increase in the habit formation parameter β will always decrease the ratio of the standard derivation of consumption growth and wealth growth. The more important the recent past consumption, the more smoother the current consumption. Without habit formation ($\beta = 0$), we obtain

$$\eta_{\beta=0} = \epsilon\rho + (1-\epsilon)\left[r + \lambda + \frac{(\mu-r)^2}{2(R+\kappa)\sigma^2}\right] - \lambda.$$

Thus we have $\eta < \eta_{\beta=0}$, therefore habit formation can interpret the consumption smoothing puzzle.

The effects of the spirit of capitalism on the ratio of the standard derivation of consumption growth and wealth growth is given by

$$\frac{\partial \eta}{\partial \lambda} = -\frac{\epsilon(r+\lambda) + (1+\epsilon)\beta k_1\lambda}{2\beta k_1\lambda + r + \lambda} < 0.$$

An increase in the spirit of capitalism will always decrease the ratio of the standard derivation of consumption growth and wealth growth. With a strong spirit of capitalism, the investor cares more about his social status and the power of wealth, and will decrease consumption and accumulate more wealth in order to improve his social status, then consumption is more smoother. Without the spirit of capitalism ($\lambda = 0$), then we obtain $k_1 = 1/r$ and

$$\eta_{\lambda=0} = \frac{1}{1+\beta/r}\left\{\epsilon\rho + (1-\epsilon)\left[r + \frac{(\mu-r)^2}{2(R+\kappa)\sigma^2}\right]\right\}.$$

This outcome is similar to Constantinides (1990) without the spirit of capitalism, recursive utility and robustness. Since $\eta < \eta_{\lambda=0}$, the spirit of capitalism can interpret the consumption smoothing puzzle.

The effects of recursive utility on the ratio of the standard derivation of consumption growth and wealth growth is characterized as follows

$$\frac{\partial \eta}{\partial \epsilon} = \frac{1}{1+\beta k_1}\left\{\rho - \left[r + (1+\beta k_1)\lambda + \frac{(\mu - r)^2}{2(r+\kappa)\sigma^2}\right]\right\}.$$

Reasonable parameters mean $\rho - \left[r + (1+\beta k_1)\lambda + \frac{(\mu-r)^2}{2R\sigma^2}\right] < 0$, then $\frac{\partial \eta}{\partial \epsilon} < 0$, in that a higher elasticity of intertemporal substitution induces the investor to consume lower than past and decreases the ratio of the standard derivation of consumption growth and wealth growth. When $\epsilon = 1/R$, we have the ratio under CRRA utility

$$\eta_{\epsilon=1/R} = \frac{1}{(1+\beta k_1)R}\left\{\rho - (1-R)\left[r + (1+\beta k_1)\lambda + \frac{(\mu - r)^2}{2(R+\kappa)\sigma^2}\right]\right\} - \lambda.$$

This means that recursive utility can interpret the consumption smoothing puzzle with $\epsilon > 1/R$.

The effects of robustness on the ratio of the standard derivation of consumption growth and wealth growth is given by

$$\frac{\partial \eta}{\partial \kappa} = -\frac{(1-\epsilon)(\mu - r)^2}{2(1+\beta k_1)(R+\kappa)^2\sigma^2} < 0.$$

The ratio of the standard derivation of consumption growth and wealth growth decreases with uncertainty aversion. The higher the degree of uncertainty aversion of the investor, the lower the ratio of the standard derivation of consumption growth and wealth growth. When $\kappa = 0$, we obtain the ratio without robustness

$$\eta_{\kappa=0} = \frac{1}{1+\beta k_1}\left\{\epsilon\rho + (1-\epsilon)\left[r + (1+\beta k_1)\lambda + \frac{(\mu - r)^2}{2R\sigma^2}\right]\right\} - \lambda.$$

Robustness can better interpret the consumption smoothing puzzle than the case without robustness.

5. ASSET PRICING AND THE EQUITY PREMIUM PUZZLE

In this section, we will discuss how habit formation, the spirit of capitalism, recursive utility and robustness affect asset pricing. We first give the equilibrium asset pricing relationships, and then try to interpret the equity premium puzzle.

THEOREM 3. *With habit formation, the spirit of capitalism, recursive utility and robustness, the equilibrium risk premium on the risky asset must satisfy*

$$\begin{aligned}\mu - r &= \left(1+\frac{\kappa}{R}\right)\frac{1}{\epsilon}\left(1-\frac{\lambda}{h}\right)(1+\beta k_1)\frac{c^*}{c^*-(1-k_1\lambda)z}\sigma_{Pc^*} \\ &+ \left(1+\frac{\kappa}{R}\right)\left[R-\frac{1}{\epsilon}\left(1-\frac{\lambda}{h}\right)(1+\beta k_1)\right]\frac{W}{W-k_1 z}\sigma_{PW}, \quad (23)\end{aligned}$$

where $\sigma_{Pc^*}, \sigma_{PW}$ are the covariances of the risky asset's return with the investor's consumption growth and his wealth growth respectively, that is

$$\sigma_{Pc^*}dt = cov_t\left(\frac{dP}{P},\frac{dc^*}{c^*}\right), \quad \sigma_{PW}dt = cov_t\left(\frac{dP}{P},\frac{dW}{W}\right),$$

with $cov_t(\cdot,\cdot)$ being the conditional covariance operator.

Proof. See Appendix C. ▌

Equation (23) implies that consumption risk is not the only risk that should be compensated for in equilibrium. Instead, the expected risk premium for a risky asset is determined by its covariation with each investor's consumption and wealth. Intuitively, when investors care about social status, they will hedge not only against future consumption uncertainty but also against those factors that affect their future status. Since one's social status is determined by his own wealth, risk that is correlated with the variable should be compensated for. With robustness, both market risk and model uncertainty are priced in equilibrium.

To resolve the equity premium puzzle, we need compute the effective RRA coefficient in our model.

By Constantinides (1990), the effective RRA coefficient is defined by the outcome of an atemporal gamble that changes the current level of wealth. From (20), the effective RRA coefficient, ERRA, is given by

$$\text{ERRA} = -\frac{WV_{WW}}{V_W} = \frac{R}{1-k_1 z/W} = R\left[1+\frac{k_1(h-\lambda)\xi}{1-(1-k_1\lambda)\xi}\right] \geq R.$$

The effective RRA coefficient is larger than the nominal RRA coefficient R. Constantinides (1990) argues that a lower effective RRA coefficient means the equity premium puzzle is resolved.

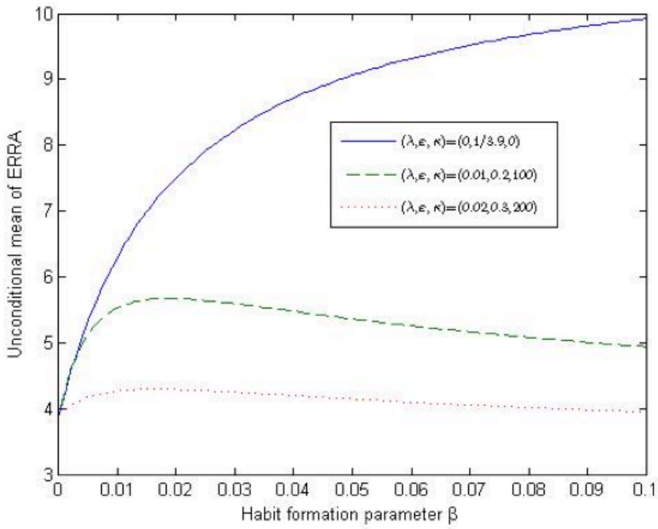
Since ξ has a steady-state distribution, the effective RRA coefficient also has a stationary distribution. From Theorem 2, for $M > 2$ we can obtain the unconditional mean of the effective RRA coefficient as follows

$$\begin{aligned} E[\text{ERRA}] &= R\left[1 + k_1(h-\lambda)E\left(\frac{\xi}{1-(1-k_1\lambda)\xi}\right)\right] \\ &= R\left[1 + k_1(h-\lambda)\left(\frac{N}{M-2}\right)\right] \geq R. \end{aligned}$$

Because the effects on $E[\text{ERRA}]$ of habit formation, the spirit of capitalism, recursive utility and robustness are very complicated, we use the numerical method to research these effects.

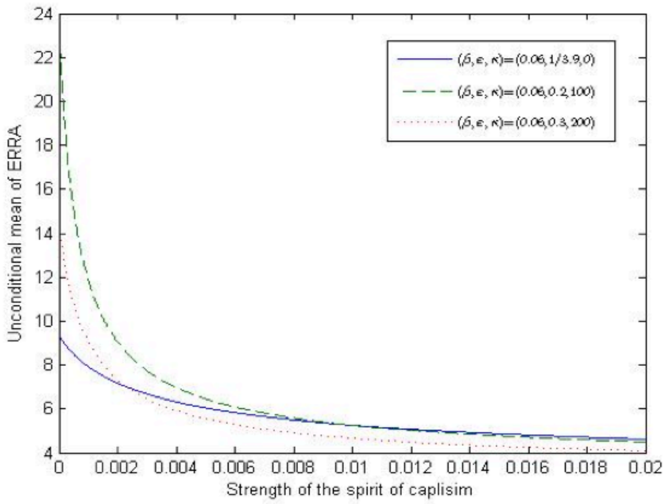
We adopt the corresponding basic parameter values in Masten (2003) for North America, which means that $\mu = 0.073$, $\sigma = 0.159$, $r = 0.02$, $R = 3.9$ and $\rho = 0.005$. The given parameters must satisfy our model conditions: $\lambda < h < 1/k_1$ and $M > 2$, so parameters β, λ, ϵ and κ have some special regions. Refer to Masten (2003) for β and Maenhout (2004) for κ, we consider the following parameter regions: $\beta \in [0, 0.1]$, $\lambda \in [0, 0.02]$, $\epsilon \in [0.15, 0.35]$ and $\kappa \in [0, 200]$. When $\beta = 0.06$, $\lambda = 0$, $\epsilon = 1/3.9$ and $\kappa = 0$, our model reduces to the case of Masten (2003) and we find $E[\text{ERRA}] = 9.3183$. $E[\text{ERRA}]$ is so high and the equity premium puzzle still remains even Masten (2003) has introduced habit formation. Now we consider the effects on $E[\text{ERRA}]$ of habit formation, the spirit of capitalism, recursive utility and robustness.

FIG. 1. Effects of habit formation on $E[\text{ERRA}]$.



The solid line in Figure 1 represents the similar model of Masten (2003) which only consider habit formation. The unconditional mean of ERRA increases with the habit formation parameter β, which means habit formation cannot interpret the equity premium puzzle. Compared with Constantinides (1990), this is an inverse conclusion between habit formation and the equity premium puzzle. The reason is that Constantinides (1990) sets habit formation as $dz(t) = (bc(t) - az(t))dt$ and E[ERRA] changes oppositely with parameters a and b, while we set $a = b = \beta$ and $dz(t) = \beta(c(t) - z(t))dt$. So for some special values of other model parameters E[ERRA] will increase with β, for example our model settings. When we introduce the spirit of capitalism, recursive utility and robustness, dot dash line and dash line indicate habit formation still cannot interpret the equity premium puzzle, but we can obtain lower E[ERRA] than Masten (2003).

FIG. 2. Effects of the spirit of capitalism on E[ERRA].



The solid line in Figure 2 represents the model that we introduce the spirit of capitalism into Masten (2003). All lines indicate that E[ERRA] decreases with the strength of the spirit of capitalism, so the spirit of capitalism is favorable to resolve the equity premium puzzle if we choose higher λ as in Bakshi and Chen (1996).

The solid line in Figure 3 represents the model that we introduce recursive utility into Masten (2003). Likewise, All lines indicate that E[ERRA] decreases with the elasticity of intertemporal substitution. If we set $\epsilon > 1/3.9$, then the equity premium puzzle can be resolved.

FIG. 3. Effects of recursive utility on E[ERRA].

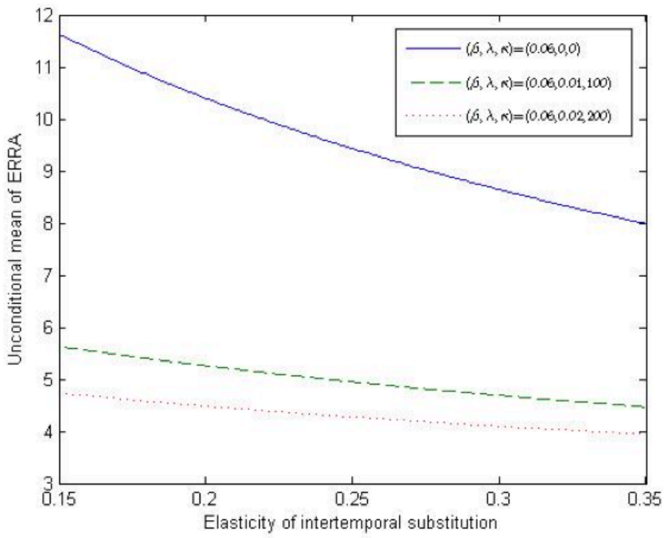
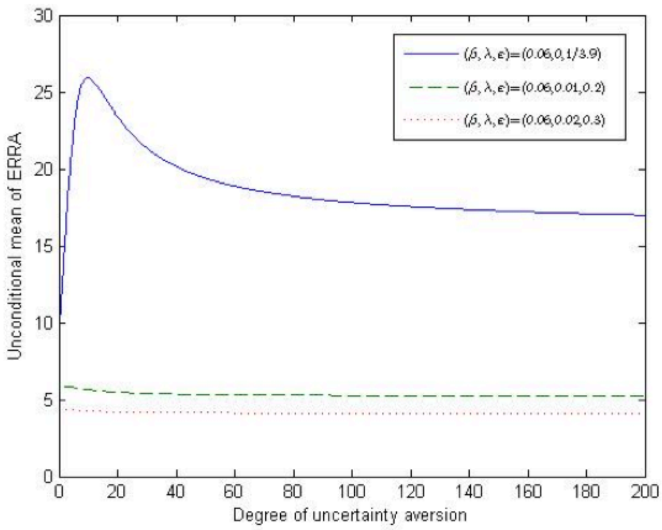


FIG. 4. Effects of robustness on E[ERRA].



The solid line in Figure 4 represents the model that we introduce robustness into Masten (2003). It shows that only robustness cannot interpret equity premium puzzle based on Masten (2003). However, if we add the spirit of capitalism and recursive utility, robustness can interpret equity premium puzzle as the dot dash line and dash line show. An increase in the degree of uncertainty aversion will decrease the effective risk aversion E[ERRA], in that uncertainty aversion is beneficial to interpret the equity premium puzzle.

In a word, the four figures shows that we can obtain the relatively lower effective RRA coefficient in our model than 9.3183 in Masten (2003), then our model can better interpret the equity premium puzzle.

6. CONCLUSION

We study the roles of habit formation, the spirit of capitalism, recursive utility and robustness in the optimal consumption and portfolio choice problem for investors. In a continuous-time stochastic model we obtain the explicit solutions of the robust consumption and portfolio choice problem, then we discuss the implications of the four factors for consumption and portfolio choice, and next we give the dynamics of consumption and the formula of asset pricing. With habit formation, the spirit of capitalism, recursive utility and robustness, we can better explain the consumption smoothing puzzle and the equity premium puzzle. However, our paper does not consider the implications of labor input on consumption and portfolio choice. With the presence of labor input, optimal consumption and optimal portfolio composition will be more complicated. Furthermore, one can examine how labor input changes consumption and portfolio selection following Gomes and Michaelides (2003), Cocco et al. (2005), Polkovnichenko (2007) and Wang (2009). Another interest extension is to examine how incomplete information affects consumption and portfolio choice following Xia (2001), Brennan and Xia (2001), Honda (2003), Lundtofte (2006, 2008), David (2008), Miao (2009), Liu (2011) and Branger et al. (2013).

APPENDIX A

Proof of Theorem 1: Substituting the first-order optimality conditions (13), (14) and (15) back into (11) and (12), we obtain the equilibrium HJB equation

$$
\begin{aligned}
&\rho V(W,z) \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(A.1)}\\
= &\frac{1/\epsilon}{1-R}(V_W-\beta V_z)^{1-\epsilon}[(1-R)V(W,z)]^{\frac{1-R\epsilon}{1-R}}+\frac{1-1/\epsilon}{1-R}\frac{(\mu-r)^2}{2\sigma^2}\frac{\kappa(1-R)V}{\left[\frac{(1-R)VV_{WW}}{V_W^2}-\kappa\right]^2}\\
+ &\frac{1-1/\epsilon}{1-R}\left\{(rW-z)V_W+\lambda W(V_W-\beta V_z)-\frac{(\mu-r)^2}{2\sigma^2}\frac{V_W^2}{V_{WW}}\frac{1}{\left[1-\frac{\kappa V_W^2}{(1-R)VV_{WW}}\right]^2}\right\}.
\end{aligned}
$$

Analogy with the standard CRRA utility, one can guess a functional form for the value function looks like

$$
V(W(t),z(t))=\frac{k_0^{\frac{1-R}{1-\epsilon}}(W(t)-k_1z(t))^{1-R}}{1-R}, \tag{A.2}
$$

where k_0, k_1 are constants to be determined. Substituting (A.2) into (A.1) yields to

$$
\begin{aligned}
&\rho(W-k_1z) \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(A.3)}\\
= &\frac{1}{\epsilon}(1+\beta k_1)^{1-\epsilon}k_0(W-k_1z)+\left(1-\frac{1}{\epsilon}\right)\frac{(\mu-r)^2}{2\sigma^2}\frac{\kappa}{(R+\kappa)^2}(W-k_1z)\\
+ &\left(1-\frac{1}{\epsilon}\right)\left\{(rW-z)+\lambda W(1+\beta k_1)+\frac{(\mu-r)^2}{2\sigma^2}\frac{R}{(R+\kappa)^2}(W-k_1z)\right\}.
\end{aligned}
$$

Setting the coefficients on W and z equal to zero in (A.3), we can obtain k_0, k_1 in Theorem 1. Substituting the value function (A.2) into the first-order optimality conditions (13), (14) and (15) yields to the optimal distort rule (17), the optimal consumption rule (18) and the optimal portfolio rule (19). For the economic sense, we require the following admissible conditions

$$
h-\lambda>0,\quad 1-hk_1>0,\quad W(t)-k_1z(t)>0.
$$

Theses inequalities imply the the admissible conditions in Theorem 1. ▮

APPENDIX B

Proof of Theorem 2:

(I) From (7) and (4), in equilibrium we can get

$$
\begin{aligned}
d(W-k_1z) &= dW-k_1dz\\
&= \{[\theta^*(\mu-r)+r]W-c^*+(\theta^*)^2\sigma^2W^2u^*-\beta k_1(c^*-z)\}dt\\
&+ \theta^*W\sigma dB.
\end{aligned}
$$

Substituting in for u^*, c^* and θ^* from (17), (18) and (19), we obtain

$$d(W - k_1 z) = (W - k_1 z)(ndt + m\sigma dB), \tag{B.1}$$

where m, n is given in (22). In terms of (18), in equilibrium one can obtain

$$\frac{d(c - z + \lambda W)}{c - z + \lambda W} = \frac{d(W - k_1 z)}{W - k_1 z} = ndt + m\sigma dB. \tag{B.2}$$

From (18), we can also get $W = \frac{1}{h-\lambda}[c-(1-hk_1)z]$. Using it we can obtain the dynamics of consumption by (18):

$$\begin{aligned}
\frac{dc}{c} &= (1 - k_1\lambda)\left[\frac{dz}{c}\right] + \left[1 - (1 - k_1\lambda)\left(\frac{z}{c}\right)\right]\left[\frac{d(W - k_1 z)}{W - k_1 z}\right] \\
&= \beta(1 - k_1\lambda)\left[1 - \left(\frac{z}{c}\right)\right] dt + \left[1 - (1 - k_1\lambda)\left(\frac{z}{c}\right)\right](ndt + m\sigma dB) \\
&= \left\{(\beta + n)\left[1 - (1 - k_1\lambda)\left(\frac{z}{c}\right)\right] - \beta k_1\lambda\right\} dt + \left[1 - (1 - k_1\lambda)\left(\frac{z}{c}\right)\right] m\sigma dB.
\end{aligned}$$

Let $\xi = z/c$, we get the equilibrium consumption dynamics (21). The composite consumption is positive, in that $c - z + \lambda W > 0$. Using (18) we can obtain $0 \leq \xi < \frac{1}{1-k_1\lambda}$.

(II) Now we consider the dynamics of ξ. In terms of Itôs lemma, we have

$$\begin{aligned}
d\xi &= d(z/c) = \frac{dz}{c} - \left(\frac{z}{c}\right)\left(\frac{dc}{c}\right) + \left(\frac{z}{c}\right)\left(\frac{dc}{c}\right)^2 \\
&= [1 - (1 - k_1\lambda)\xi]\{\beta - (\beta + n)\xi + \xi[1 - (1 - k_1\lambda)\xi]m^2\sigma^2\}dt \\
&\quad -\xi[1 - (1 - k_1\lambda)\xi]m\sigma dB.
\end{aligned}$$

Let $\zeta = \frac{\xi}{1-(1-k_1\lambda)\xi}$, then $\zeta \geq 0$ and

$$\begin{aligned}
d\zeta &= d\left[\frac{\xi}{1 - (1 - k_1\lambda)\xi}\right] = \frac{1}{[1 - (1 - k_1\lambda)\xi]^2} d\xi + \frac{1 - k_1\lambda}{[1 - (1 - k_1\lambda)\xi]^3}(d\xi)^2 \\
&= [\beta - (n + \beta k_1\lambda - m^2\sigma^2)\zeta]dt - m\sigma\zeta dB. \qquad\qquad (\text{B.3})
\end{aligned}$$

This is an autonomous linear differential equation, then there exits an unique solution $\zeta(t)$ and $\zeta(t)$ is a homogeneous Markov process. Let $p(t, y_0, y)$ denote the transition density of $\zeta(t)$, i.e.

$$p(t, y_0, y) = \frac{d}{dt} P(\zeta(t_0 + t) \leq y | \zeta(t_0) = y_0).$$

Denote $p = p(t, y_0, y)$, then p satisfies the following Kolmogorov forward equation

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial y}\{[\beta - (n + \beta k_1\lambda - m^2\sigma^2)y]p\} + \frac{1}{2}\frac{\partial^2[m^2\sigma^2y^2\pi(y)]}{\partial y^2} = 0. \quad \text{(B.4)}$$

The limiting distribution of ζ is the unique stationary distribution. Let $\pi(y)$ denote the steady-state probability density, then $\pi(y) = p(\infty, y_0, y)$ and $\pi(y)$ satisfies (B.4), in that

$$\frac{\partial}{\partial y}\{[\beta - (n + \beta k_1\lambda - m^2\sigma^2)y]\pi(y)\} - \frac{1}{2}\frac{\partial^2[m^2\sigma^2y^2\pi(y)]}{\partial y^2} = 0. \quad \text{(B.5)}$$

The solution of (B.5) is that

$$\pi(y) = \frac{N^{M_1}}{\Gamma(M-1)}y^{-M}e^{-N/y}, \quad y \geq 0, \quad \text{(B.6)}$$

where $M = \frac{2(n+\beta k_1\lambda)}{m^2\sigma^2} > 1$, $N = \frac{2\beta}{m^2\sigma^2}$. Since $\xi = \frac{\zeta}{1+(1-k_1\lambda)\zeta}$ is a strictly monotone increasing function and $\zeta = \frac{\xi}{1-(1-k_1\lambda)\xi}$ has a continuous first-order derivative, then ξ has a stationary distribution with density

$$\begin{aligned}
\Psi(x) &= \pi\left(\frac{x}{1-(1-k_1\lambda)x}\right)\cdot\left|\frac{dy}{dx}\right| \\
&= \frac{N^{M-1}}{\Gamma(M-1)}x^{-2}\left[\frac{1-(1-k_1\lambda)x}{x}\right]^{M-2}\exp\left\{-N\left[\frac{1-(1-k_1\lambda)x}{x}\right]\right\}, \\
0 &\leq x < \frac{1}{1-k_1\lambda}.
\end{aligned}$$

This completes the proof. ▌

APPENDIX C

Proof of Theorem 3: Rewrite (14) and (15) as follows

$$\begin{aligned}
V_W &= \beta V_z + (c^* - z + \lambda W)^{-\frac{1}{\epsilon}}([1-R]V)^{\frac{1/\epsilon - R}{1-R}}, && \text{(C.1)} \\
(\mu - r)dt &= \left[1 - \Psi(W,z)f'([1-R]V)\frac{V_W^2}{V_{WW}}\right]\left[-\frac{WV_{WW}}{V_W}\right]\text{cov}\left(\frac{dP}{P}, \frac{dW}{W}\right) && \text{(C.2)}
\end{aligned}$$

Applying Itô's lemma for V_W, we have $dV_W = V_{WW}dW + V_{Wz}dz + \frac{1}{2}V_{WWW}(dW)^2$, therefore

$$\text{cov}\left(\frac{dP}{P}, \frac{dV_W}{V_W}\right) = \left[\frac{WV_{WW}}{V_W}\right]\text{cov}\left(\frac{dP}{P}, \frac{dW}{W}\right).$$

Then we can rewrite (C.2) as that

$$\begin{aligned}(\mu - r)dt &= -\left[1 - \Psi(W,z)f'([1-R]V)\frac{V_W^2}{V_{WW}}\right]\mathrm{cov}\left(\frac{dP}{P},\frac{dV_W}{V_W}\right) \\ &= -\left(1+\frac{\kappa}{R}\right)\mathrm{cov}\left(\frac{dP}{P},\frac{dV_W}{V_W}\right). \qquad \text{(C.3)}\end{aligned}$$

On the other hand, using (C.1) we can get

$$\begin{aligned}dV_W &= \beta dV_z + (1-R)^{frac1/\epsilon - R1-R} d\left((c^* - z + \lambda W)^{-\frac{1}{\epsilon}} \cdot V^{\frac{1/\epsilon - R}{1-R}}\right) \\ &= \beta dV_z + (1-R)^{\frac{1/\epsilon - R}{1-R}}\Big[V^{\frac{1/\epsilon-R}{1-R}} \cdot d(c^* - z + \lambda W)^{-\frac{1}{\epsilon}} \\ &+ \ (c^* - z + \lambda)^{-\frac{1}{\epsilon}} \cdot dV^{\frac{1/\epsilon-R}{1-R}} + d(c^* - z + \lambda W)^{-\frac{1}{\epsilon}} \cdot dV^{\frac{1/\epsilon-R}{1-R}}\Big],\end{aligned}$$

where

$$\begin{aligned}dV_z &= V_{zW}dW + V_{zz}dz + \frac{1}{2}V_{zWW}(dW)^2, \\ d(c^* - z + \lambda W)^{-\frac{1}{\epsilon}} &= -\frac{1}{\epsilon}(c^* - z + \lambda W)^{-\frac{1}{\epsilon}-1}(dc^* - dz + \lambda dW) \\ &+ \frac{1}{2}\frac{1}{\epsilon}\left(\frac{1}{\epsilon}+1\right)(c^* - z + \lambda W)^{-\frac{1}{\epsilon}-2}(dC^* - dz + \lambda dW)^2, \\ dV^{\frac{1/\epsilon-R}{1-R}} &= \frac{1/\epsilon - R}{1-R}V^{\frac{1/\epsilon-R}{1-R}-1}dV + \frac{1}{2}\frac{1/\epsilon-R}{1-R}\left(\frac{1/\epsilon-R}{1-R}-1\right)V^{\frac{1/\epsilon-R}{1-R}-2}(dV)^2 \\ &= \frac{1/\epsilon-R}{1-R}V^{\frac{1/\epsilon-R}{1-R}-1}\left[V_W dW + V_z dz + \frac{1}{2}V_{WW}(dW)^2\right] \\ &+ \frac{1}{2}\frac{1/\epsilon-R}{1-R}\left(\frac{1/\epsilon-R}{1-R}-1\right)V^{\frac{1/\epsilon-R}{1-R}-2}\left[V_W dW + V_z dz + \frac{1}{2}V_{WW}(dW)^2\right]^2.\end{aligned}$$

Thus we can obtain

$$\begin{aligned}\mathrm{cov}\left(\frac{dP}{P},\frac{dV_W}{V_W}\right) &= \frac{\beta W V_{zW}}{V_W}\mathrm{cov}\left(\frac{dP}{P},\frac{dW}{W}\right) \\ &- \frac{1}{\epsilon}\frac{V_W - \beta V_z}{V_W}\frac{c^*}{c^* - z + \lambda W}\mathrm{cov}\left(\frac{dP}{P},\frac{dc^*}{c^*}\right) \\ &- \frac{\lambda}{\epsilon}\frac{V_W - \beta V_z}{V_W}\frac{W}{c^* - z + \lambda W}\mathrm{cov}\left(\frac{dP}{P},\frac{dW}{W}\right) \\ &+ \frac{1/\epsilon - R}{1-R}\frac{W(V_W - \beta V_z)}{V}\mathrm{cov}\left(\frac{dP}{P},\frac{dW}{W}\right) \\ &= -\frac{1}{\epsilon}\left(1-\frac{\lambda}{h}\right)(1+\beta k_1)\frac{c^*}{c^* - (1-\lambda k_1)z}\mathrm{cov}\left(\frac{dP}{P},\frac{dc^*}{c^*}\right) \\ &- \left[R - \frac{1}{\epsilon}\left(1-\frac{\lambda}{h}\right)(1+\beta k_1)\right]\frac{W}{W - k_1 z}\mathrm{cov}\left(\frac{dP}{P},\frac{dW}{W}\right).\end{aligned}$$

Substituting it into (C.3) and let $\sigma_{Pc^*}dt = \text{cov}\left(\frac{dP}{P}, \frac{dc^*}{c^*}\right)$ and $\sigma_{PW}dt = \text{cov}\left(\frac{dP}{P}, \frac{dW}{W}\right)$, we have the asset pricing formula (23). ▌

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