

Institution/Transaction Costs and Economic Growth

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This paper develops a dynamic economic growth model in which institutions are treated as real cost technologies borne directly by households that produce, consume, and own firms. Building on Coase's insight that institutions exist to economize on the costs of exchange, Williamson's theory of comparative governance, North's historical thesis on institutional divergence, and Cheung's claim that transaction costs are institution costs, we construct a framework where governance structures absorb resources and alter intertemporal incentives. In this setting, the representative capitalist household faces not only the usual trade-off between consumption and saving but also the resource drain and dynamic wedges created by supervision, verification, and bureaucratic complexity. Institutions thus determine both the feasibility of exchange and the trajectory of growth.

The analysis demonstrates how poor institutions depress capital accumulation and consumption, widen the gap between potential and realized outcomes, and lock economies into low-level equilibria. Reforms that reduce verification costs deliver immediate consumption gains, while reductions in supervision and complexity costs require transitional sacrifices but yield higher long-run capital and welfare. Comparative governance is formalized as a boundary condition that selects markets, firms, or hybrids according to which minimizes institution costs at the relevant scale. When governance intensity both raises productivity and consumes resources, institutional performance follows an inverted-U pattern, generating a Laffer-type curve. The model shows that institutional change is the hinge of capitalist development: societies that reduce institution costs achieve sustained growth, while those that do not stagnate.

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1. INTRODUCTION

The theory of economic growth has long been dominated by models that treat institutions as background conditions or exogenous parameters, while assuming that a benevolent planner directs the allocation of resources. Such

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formulations, though analytically convenient, obscure the essential fact that capitalist economies are not managed by planners but by households and firms who own resources, bear costs, and make decentralized decisions. Institutions in this context are not external constraints imposed from above but endogenous arrangements that enable exchange at a cost. To model growth without placing the representative capitalist household at the center is to miss what Coase (1937) identified as the fundamental problem: why institutions exist in the first place.

In his classic essay on the nature of the firm, Coase argued that market exchange is costly and that firms arise to economize on the use of the price mechanism. Williamson (1975, 1985) extended this insight into a comparative governance framework, showing that markets, hierarchies, and hybrids are alternative organizational forms whose relative efficiency depends on the attributes of transactions. North (1990, 2005) shifted attention to history, arguing that the long-run divergence of societies can be traced to the institutions that reduce or perpetuate the costs of exchange. Cheung (1998) crystallized these insights into a single claim: transaction costs are institution costs. These are not abstract frictions but real resources consumed in supervision, verification, and bureaucratic complexity, borne by households and firms in order to make trade possible.

This paper develops a dynamic general equilibrium model that embeds these insights directly into the budget constraint and intertemporal choices of a representative capitalist household. The household produces output, consumes goods, accumulates capital, and owns firms, but must also pay for the institutions that enable production and exchange. Institutions are represented as a cost technology that absorbs resources and alters incentives, thereby shaping both the feasible set and the growth trajectory. By moving away from planner formulations, the analysis captures the reality of capitalist economies in which private actors choose among governance structures, bear their costs, and reap their returns.

The contribution of this framework is threefold. First, it formalizes Coase's efficiency principle by showing how the welfare effect of institutions is exactly the shadow-valued present value of resources they consume. Second, it provides a quantitative boundary-of-the-firm condition that captures Williamson's comparative governance logic: firms emerge when supervision and complexity costs are lower than verification costs in markets, and markets dominate when the reverse is true. Third, it demonstrates that North's thesis of institutional divergence has a precise dynamic counterpart: societies that reduce institution costs converge to high-capital steady states, while those that fail stagnate. By treating transaction costs as institution costs, as Cheung insisted, the model ensures that institutional economics is not peripheral but central to the theory of growth.

This approach also connects with more recent contributions to the literature on institutions and development. Acemoglu, Johnson, and Robinson (2001) and Acemoglu and Robinson (2012) argue that political institutions determine long-run prosperity by shaping incentives and property rights, a perspective consistent with our focus on supervision, verification, and complexity costs. Greif (2006) demonstrates how historical institutional arrangements in medieval trade reduced contracting costs, offering case studies that can be interpreted through our cost technology. Nelson and Winter's (1982) evolutionary theory of economic change highlights how routines and organizational forms evolve as cost-saving devices, complementing the Coasean logic of institution formation. By embedding these institutional considerations in the structure of the Ramsey–Cass–Koopmans model, this paper provides a bridge between classical institutional economics and modern growth theory, unifying historical, contractual, and evolutionary perspectives under a common analytic framework.

The remainder of the paper is organized as follows. Section 2 sets up the decentralized capitalist household model, embedding institution costs directly in its budget constraint and optimization problem. Section 3 derives the dynamic Euler equation and steady-state conditions under alternative governance structures. Section 4 provides a comparative static analysis of the institutional parameters, showing how supervision, verification, and complexity costs affect capital, consumption, and welfare. Section 5 examines policy and reform experiments, including transitional dynamics and the institutional Laffer curve. Section 6 develops the boundary-of-the-firm condition, linking Coase and Williamson within the dynamic system. Section 7 explores institutional change over time and its implications for long-run divergence, building on North's historical thesis. Section 8 concludes by synthesizing the results and highlighting the central role of institutions as real cost technologies in capitalist economies.

2. MODEL SETUP

In order to capture the functioning of a capitalist economy, it is essential to avoid recourse to a benevolent planner's problem. As Coase (1937, 1960) emphasized, institutions exist not because a social planner optimizes aggregate welfare, but because private agents, operating within markets, seek to economize on the costs of carrying out exchange. Williamson (1975, 1985) similarly stressed that governance structures are chosen by firms and individuals as comparative contractual devices rather than imposed from above. North (1990, 2005) argued that institutional change is path dependent, reflecting the incentives of political and economic actors, while Cheung (1998) insisted that transaction costs are nothing but institution costs, borne directly by real agents. Building on this perspective, we construct

a model in which a representative capitalist household simultaneously produces, consumes, saves, and bears the costs of maintaining institutions.

The household derives utility from consumption according to a standard intertemporal utility function consistent with the Ramsey–Cass–Koopmans tradition. Preferences are represented by

$$U = \int_0^\infty e^{-\rho t} \frac{c(t)^{1-\sigma} - 1}{1-\sigma} dt,$$

where $c(t)$ denotes consumption at time t , $\rho > 0$ is the subjective discount rate, and $\sigma > 0$ is the coefficient of relative risk aversion. This form ensures comparability with canonical growth theory, but in our context it describes the valuation of the consumption stream by the very household that also owns capital and firms.

Production is carried out through firms that are owned by the household, and aggregate output follows a neoclassical production function of the form

$$y(t) = F(k(t)) = Ak(t)^\alpha, \quad 0 < \alpha < 1,$$

with $A > 0$ representing total factor productivity and α the capital share. Capital depreciates at rate $\delta \geq 0$. Because firms are owned by the representative household, profits are distributed directly back to the household, ensuring consistency with capitalist ownership rather than centralized planning.

Institutions are modeled as technologies that absorb real resources. Following Cheung's (1998) equivalence, transaction costs are defined as institution costs and are explicitly deducted from the household's feasible allocation. The institutional cost function is given by

$$\Phi(k, c; i) = \gamma(i)k + \phi(i)c + \frac{1}{2}\eta(i)k^2,$$

where i indexes the chosen governance structure, such as market exchange, hierarchical firm organization, or hybrid arrangements. The parameter $\gamma(i)$ reflects supervision and monitoring costs, consistent with the analysis of Alchian and Demsetz (1972) and Williamson (1985). The parameter $\phi(i)$ measures verification and enforcement costs in contracting and delivery, in the spirit of Barzel (1997). Finally, $\eta(i)$ captures the increasing complexity of bureaucratic administration as the scale of production expands, a phenomenon emphasized by both Williamson and North. These terms represent resources that must be consumed in order to make transactions feasible; they are not mere welfare losses but real goods and services absorbed in the operation of institutions.

Capital accumulation for the household therefore takes the form

$$\dot{k}(t) = Ak(t)^\alpha - (\delta + \gamma(i))k(t) - \frac{1}{2}\eta(i)k(t)^2 - (1 + \phi(i))c(t),$$

which makes clear that institution costs directly reduce the resources available for saving and consumption. This expression embodies Coase's insight that using the price system consumes resources, and Williamson's logic that alternative governance arrangements entail different institutional burdens.

The household chooses its consumption path in order to maximize utility subject to this accumulation constraint. The Hamiltonian associated with this private optimization problem is

$$\mathcal{H}(c, k; \lambda) = \frac{c^{1-\sigma} - 1}{1 - \sigma} + \lambda \left(F(k) - \delta k - c - \Phi(k, c; i) \right),$$

where λ is the co-state variable interpreted as the private shadow value of capital. The first-order conditions are straightforward. The condition for consumption is

$$c^{-\sigma} = \lambda(1 + \phi(i)),$$

which shows that marginal utility is scaled by the verification wedge. The costate equation is

$$\dot{\lambda} = \lambda \left(\rho + \delta + \gamma(i) + \eta(i)k - F'(k) \right),$$

which makes clear that institution costs raise the user cost of capital. The state equation repeats the accumulation constraint, and the Euler equation obtained by combining the conditions is

$$\frac{\dot{c}}{c} = \frac{1}{\sigma} \left(F'(k) - \delta - \rho - \gamma(i) - \eta(i)k \right).$$

In this decentralized setting, institution costs appear twice: as a resource drain that lowers feasible output and consumption, and as a dynamic wedge that reduces the effective return on capital. This dual role makes precise Cheung's (1998) claim that transaction costs are institution costs. It also embodies Coase's (1937) explanation for the emergence of firms, since the household chooses between alternative institutional regimes characterized by different (γ, ϕ, η) . Williamson's comparative governance becomes explicit in the trade-offs between market and hierarchy: markets economize on supervision but require high verification, whereas firms economize on verification but impose higher supervision and complexity. Finally, North's historical thesis appears in the comparative statics: societies with high institution costs converge to low steady states of capital and consumption, while those that reduce (γ, ϕ, η) converge to higher levels of development.

This reformulation ensures that institutions are embedded directly into the budget constraint of the representative capitalist household, rather than

into a planner's objective function. The household is the producer, consumer, and owner of firms, and bears the costs of institutions directly. In this way, the model remains firmly anchored in the logic of a capitalist economy while giving mathematical precision to the insights of Coase, Williamson, North, and Cheung.

3. DYNAMICS AND EULER EQUATION WITH INSTITUTIONAL WEDGES

Having established that the representative capitalist household owns firms, consumes, and bears institution costs directly, the dynamics of the system can be derived from the household's optimality conditions. The institutional cost function enters the problem in two ways: it subtracts directly from available resources and it alters the effective marginal return on capital. This dual role formalizes Coase's insight that institutions are chosen to economize on the cost of carrying out transactions, while simultaneously embedding Cheung's identity that transaction costs are institution costs.

The first-order condition for consumption equates marginal utility with the shadow value of resources adjusted by the verification wedge:

$$c^{-\sigma} = \lambda(1 + \phi(i)).$$

This condition makes explicit that consumption is more expensive at the margin when institutions impose verification costs. In Barzel's (1997) terms, property rights enforcement and contracting introduce a wedge between the utility derived from goods and the effective shadow value of delivering them. The shadow value of capital, λ , thus reflects not only intertemporal substitution but also the institutional burden associated with consumption.

The dynamic condition for the co-state variable, which represents the household's valuation of capital, is

$$\dot{\lambda} = \lambda(\rho + \delta + \gamma(i) + \eta(i)k - F'(k)).$$

This expression highlights that the user cost of capital is raised by both constant supervision costs, γ , and scale-dependent complexity costs, ηk . In Williamson's framework of governance, these correspond to the bureaucratic burdens associated with internal organization or state regulation, which raise the effective hurdle rate for investment. The return to capital is not merely $F'(k) - \delta$ but is reduced further by institutional costs, showing how governance structures alter incentives at the margin.

Capital accumulation itself is given by the household's budget constraint,

$$\dot{k} = Ak^\alpha - (\delta + \gamma(i))k - \frac{1}{2}\eta(i)k^2 - (1 + \phi(i))c,$$

which records the resources available after consumption and institutional costs are paid. The three institutional parameters operate differently: γ acts like an additional depreciation rate, reducing net capital regardless of scale; η introduces a quadratic congestion term, penalizing large-scale accumulation; and ϕ inflates the resource cost of each unit of consumption, reducing the level of goods that can be enjoyed out of output. This differentiation is consistent with Williamson's discriminating alignment, where different governance structures economize differently depending on the type of transaction and the scale at which it is organized.

Combining these conditions yields the Euler equation for consumption dynamics,

$$\frac{\dot{c}}{c} = \frac{1}{\sigma} \left(F'(k) - \delta - \rho - \gamma(i) - \eta(i)k \right).$$

This equation makes fully explicit the role of institutions in dynamic accumulation. In the standard Ramsey model, growth of consumption depends on the difference between the marginal product of capital and the sum of depreciation and the rate of time preference. Here, the same condition is modified by institutional wedges: supervision costs γ and complexity costs ηk reduce the effective return on capital, lowering the growth rate of consumption. Coase's observation that institutions exist to minimize the costs of exchange is thus rendered in precise dynamic form: poor institutions depress the growth rate directly.

At a steady state, both $\dot{c}$ and $\dot{k}$ vanish. The Euler condition then implies

$$\alpha A(k)^{\alpha-1} = \delta + \rho + \gamma(i) + \eta(i)k,$$

which determines the steady-state capital stock. This generalizes the familiar Ramsey condition by including institutional depreciation, γ , and scale-dependent complexity, ηk . The level of steady-state capital is therefore smaller when supervision or complexity costs are high, a direct formalization of North's thesis that poor institutions constrain the long-run accumulation path. The corresponding steady-state level of consumption is

$$c = \frac{1}{1 + \phi(i)} \left(A(k)^\alpha - (\delta + \gamma(i))k - \frac{1}{2}\eta(i)(k)^2 \right).$$

This shows that verification costs, ϕ , scale down feasible consumption uniformly, while supervision and complexity costs not only drain resources but also reduce the steady-state level of capital.

The dynamics around the steady state are governed by the linearization of the system. The Jacobian matrix has one positive and one negative eigenvalue, ensuring saddle-path stability.¹ The slope of the stable manifold through the steady state, which determines how consumption must adjust relative to capital, is altered by the institutional parameters. In particular, higher verification costs ϕ steepen the adjustment path by raising the marginal cost of consumption, while higher supervision and complexity costs reduce the level toward which the system converges. This captures North's idea of institutional path dependence: the economy converges stably, but toward a lower steady state when institutions are inefficient.

A comparison with the institutional golden rule is instructive. The level of capital that maximizes steady-state consumption satisfies

$$\alpha A(k^{GR})^{\alpha-1} = \delta + \gamma(i) + \eta(i)k^{GR},$$

which omits the rate of time preference. As in the standard Ramsey model, the actual steady state k is below the golden rule because impatience reduces saving. However, both k and k^{GR} are depressed by poor institutions, and the efficiency gap between them widens as γ and η rise. This means that even if society could overcome time preference, inefficient institutions would still prevent it from achieving higher potential consumption. The model thus provides a formal counterpart to North's claim that institutional inefficiencies impose long-run constraints on development.

In summary, the Euler equation and steady-state analysis demonstrate how institutions enter the dynamics of a capitalist economy. The household's consumption and saving decisions are distorted by wedges that reflect the costs of verification, supervision, and complexity. These costs reduce the growth rate of consumption, depress steady-state capital and consumption, and widen the efficiency gap relative to the golden rule. Coase's theory of institutions as devices to economize on transaction costs, Williamson's comparative governance framework, North's emphasis on historical divergence, and Cheung's equivalence between transaction and institution costs are all unified here in a dynamic general equilibrium model of a capitalist economy in which the representative agent owns firms, consumes output, and directly bears the costs of institutions.

4. COMPARATIVE STATICS OF INSTITUTIONAL PARAMETERS

The steady-state conditions derived in the previous section allow us to examine systematically how institutional parameters influence long-run outcomes and the dynamics of adjustment. In the standard Ramsey–Cass–

¹Proposition 1 in the appendix offers a rigorous proof of this saddle-point stability.

Koopmans framework, steady-state capital is determined by the balance between the marginal product of capital, depreciation, and the rate of time preference. In our institutional extension, supervision, verification, and complexity costs enter explicitly into this balance, shifting both the steady-state level of capital and the associated level of consumption.

From the Euler equation, the steady-state capital stock satisfies

$$\alpha A(k)^{\alpha-1} = \delta + \rho + \gamma + \eta k,$$

where γ and η are the institutional cost parameters associated with supervision and scale-dependent complexity. It is immediate from this expression that both parameters reduce the steady-state capital stock. A marginal increase in supervision costs γ raises the effective user cost of capital uniformly, shifting the equality toward a lower k . An increase in complexity costs η has a scale-dependent effect, intensifying the penalty as k rises, thereby depressing the equilibrium even more strongly in large-scale economies. These comparative statics mirror Williamson's (1985) logic of hierarchical organization: as supervision and complexity costs mount, the gains from accumulation are eroded, lowering the efficient scale of the firm and the aggregate capital stock.

Consumption in steady state is determined by the resource constraint,

$$c = \frac{1}{1+\phi} \left(A(k)^\alpha - (\delta + \gamma)k - \frac{1}{2}\eta(k)^2 \right).$$

This expression shows that verification costs, represented by ϕ , enter as a pure level wedge. For any given capital stock, higher verification costs reduce feasible consumption proportionally, echoing Barzel's (1997) argument that property rights and contracting absorb a fraction of output in the form of transaction costs. By contrast, supervision and complexity costs operate in two ways: they reduce consumption directly through the resource drain terms and indirectly by lowering the steady-state capital stock. The overall effect is a systematic reduction in c .

The comparative statics can be made more precise. Differentiating the steady-state condition for k with respect to γ yields

$$\frac{\partial k}{\partial \gamma} = \frac{k}{F''(k) - \eta} < 0,$$

since the production function is concave ($F''(k) < 0$). Similarly,

$$\frac{\partial k}{\partial \eta} = \frac{1}{F''(k) - \eta} < 0.$$

Both effects confirm that higher supervision and complexity costs reduce capital accumulation.

Consumption responds as

$$\frac{\partial c}{\partial \phi} = -\frac{c}{1+\phi} < 0,$$

$$\frac{\partial c}{\partial \gamma} = -\frac{1}{1+\phi}(\rho \frac{\partial k}{\partial \gamma} - k) < 0,$$

$$\frac{\partial c}{\partial \eta} = -\frac{1}{1+\phi}(\rho \frac{\partial k}{\partial \eta} - \frac{1}{2}k^2) < 0.$$

These expressions make clear that all three institutional parameters depress steady-state consumption: verification costs reduce it directly; supervision and complexity costs reduce it both directly and indirectly through their effect on the capital stock.

Turning to dynamics, the linearized system around the steady state exhibits saddle-path stability, with one stable and one unstable root. The Jacobian matrix is given by

$$J = \begin{bmatrix} \rho & -(1-\phi) \\ \frac{c}{\sigma}(F''(k) - \eta) & 0 \end{bmatrix}.$$

Its determinant is negative because $F''(k) - \eta < 0$, which ensures the existence of a stable manifold. Comparative statics of this Jacobian reveal that higher verification costs ϕ steepen the adjustment path by magnifying the sensitivity of capital dynamics to changes in consumption. Increases in supervision and complexity costs not only shift the steady state downward but also alter the curvature of the stable path by making $F''(k) - \eta$ more negative. This implies faster convergence in formal terms but to a lower steady state of capital and consumption. The dynamic system therefore confirms North's argument that poor institutions impose not just static efficiency losses but enduring growth penalties: even as economies converge, they converge toward poorer outcomes.

Comparison with the institutional golden rule reinforces this interpretation. The golden rule capital stock satisfies

$$\alpha A(k^{GR})^{\alpha-1} = \delta + \gamma + \eta k^{GR},$$

which maximizes steady-state consumption for a patient society. The Ramsey steady state, however, includes the additional term ρ , and thus always lies below the golden rule. Both the golden rule and the actual steady state are reduced by higher supervision and complexity costs, but because impatience reduces saving further, the gap between k^* and k^{GR} widens

as institutions deteriorate. This widening efficiency gap represents a formalization of North's historical claim that institutional inefficiency not only reduces long-run potential but also makes societies systematically fall short of their productive possibilities.

The welfare implications of marginal institutional reform can be expressed through the envelope formula, which equates the welfare gain from reducing an institutional parameter with the present value of the shadow-valued resources saved. For example, the welfare effect of a marginal reduction in supervision costs is

$$\frac{dW}{d\gamma} = - \int_0^{\infty} e^{-\rho t} \lambda(t) k(t) dt < 0,$$

showing that lowering supervision costs always improves welfare by the value of the capital resources freed. Similar expressions apply to verification and complexity costs. This provides a rigorous and general statement of Coase's efficiency principle in dynamic general equilibrium: the best institution is the one that minimizes the shadow-valued resource cost of effectuating exchange.

In sum, the comparative statics of the model demonstrate that institutional parameters systematically reduce capital accumulation, consumption, and welfare, while also altering the dynamics of adjustment. Verification costs act as level wedges, supervision costs as depreciation-like burdens, and complexity costs as scale-dependent penalties. These effects embody Coase's logic of institutions as cost-saving devices, Williamson's comparative governance framework, North's thesis of institution-driven divergence, and Cheung's identification of transaction costs with institution costs. The result is a rigorous characterization of how institutions shape both the steady state and the transition path of capitalist economies.

5. POLICY AND REFORM EXPERIMENTS: TRANSITIONS, WELFARE, AND THE INSTITUTIONAL LAFFER CURVE

The dynamic framework also provides a natural laboratory for evaluating institutional reforms. Since institutions are represented explicitly as cost technologies that absorb resources and alter intertemporal margins, changing their parameters has clear effects on both the steady state and the transition path. This makes it possible to analyze not only long-run outcomes but also the short-run sacrifices or gains that reforms may entail. Such an analysis reflects Coase's (1960) principle that the efficiency of an institutional arrangement should be judged by the costs of effectuating exchange, Williamson's (1985) discriminating alignment between transactions and governance structures, and North's (1990) argument that institutional change is the central determinant of long-run development trajectories.

Suppose the economy is initially in a steady state associated with institutional parameters $(\gamma_0, \phi_0, \eta_0)$. At time zero, a permanent reform reduces one of these parameters, reflecting, for example, the introduction of better property rights enforcement, a reduction in bureaucratic monitoring, or simplification of governance procedures. Since capital is predetermined, the stock of capital at the moment of reform equals the old steady-state value k_0^* . However, the new steady state associated with the reformed parameters, say $(\gamma_1, \phi_1, \eta_1)$, will in general differ from the old one. The economy must then move along the new stable path toward the new equilibrium, with consumption adjusting immediately to place the system on the stable arm.

Consider first a reduction in verification costs, ϕ . In this case, the steady-state capital stock is unaffected, since ϕ does not enter the Euler condition, but steady-state consumption rises directly according to

$$c_1 = \frac{1}{1 + \phi_1} \left(A(k)^\alpha - (\delta + \gamma)k - \frac{1}{2}\eta(k)^2 \right),$$

which exceeds the old level c_0 . The adjustment of consumption is immediate, because capital does not need to move for the benefit to be realized. The representative household enjoys a pure level gain: more of output is available for consumption once verification costs are reduced. This corresponds to Barzel's (1997) argument that greater clarity and enforceability of property rights reduce the fraction of resources wasted in contracting, and it illustrates Cheung's (1998) point that reductions in transaction costs are equivalent to reductions in institution costs directly borne by households.

Reforms to supervision or complexity costs, by contrast, affect both capital and consumption. If γ or η is reduced, the Euler condition

$$\alpha A(k)^{\alpha-1} = \delta + \rho + \gamma + \eta k$$

implies that the new steady-state capital stock is higher. The household therefore faces an incentive to accumulate more capital over time. However, since capital is predetermined, the economy begins below the new steady state, and to move onto the new stable path, consumption must adjust downward immediately. The household sacrifices some current consumption in order to save more, generating an "investment boom" that raises capital and eventually consumption. This transitional sacrifice illustrates North's observation that institutional reforms, though efficiency-enhancing in the long run, may entail short-run costs that make them politically difficult to implement, especially when incumbents rely on rents from supervision or bureaucracy.

The transitional dynamics can be approximated linearly around the new steady state. If k_1 denotes the new capital stock, then

$$k(t) = k_1 + (k_0 - k_1)e^{\lambda_- t}, \quad c(t) = c_1 + m_-(k_0 - k_1)e^{\lambda_- t},$$

where $\lambda_- < 0$ is the stable root of the linearized system and m_- the slope of the stable arm. The adjustment is monotonic, with capital converging upward to the new higher steady state and consumption rising gradually after an initial drop. Welfare effects of such reforms can be captured by the envelope condition, which states that the gain equals the present value of shadow-valued resources saved. For instance, a reduction in supervision costs yields

$$\frac{dW}{d\gamma} = - \int_0^\infty e^{-\rho t} \lambda(t) k(t) dt < 0,$$

so that welfare rises in proportion to the capital resources released. This is a dynamic restatement of Coase's efficiency principle: the value of reform is exactly the shadow-valued resource saving it generates.

The magnitude of welfare gains from alternative reforms can be compared by evaluating the shadow-valued savings associated with each cost parameter. Near steady state, the approximate welfare value of reducing supervision is $\lambda k/\rho$, of reducing verification is $\lambda c/\rho$, and of reducing complexity is $\lambda(k)^2/(2\rho)$, where $\lambda = c^{-\sigma}/(1 + \phi)$ is the shadow value of resources. This ranking provides a sufficient statistic for prioritizing reforms: in capital-intensive economies, reducing supervision or complexity costs yields the greatest welfare gains, while in economies dominated by exchange of final goods, reducing verification costs is most beneficial. This is Williamson's discriminating alignment rendered in quantitative form: governance reforms should target the parameter that multiplies the largest shadow-weighted scale variable.

An important implication of this analysis is that institutional reform may exhibit Laffer-type properties. Governance efforts may simultaneously increase productivity and consume resources. If we let governance intensity be parameterized by s , so that $A = A(s)$ with $A'(s) > 0$ and $\gamma = \gamma(s)$, $\phi = \phi(s)$, $\eta = \eta(s)$ with positive derivatives, then welfare changes are given by

$$\frac{dW}{ds} = \int_0^\infty e^{-\rho t} \lambda(t) \left(k(t)^\alpha A'(s) - \gamma'(s) k(t) - \phi'(s) c(t) - \frac{1}{2} \eta'(s) k(t)^2 \right) dt.$$

At low levels of enforcement, increases in governance effort raise productivity faster than they raise costs, so welfare rises. At high levels, however, the marginal costs dominate, and welfare falls. The optimum lies at the point where marginal benefits equal marginal costs. This inverted-U shape represents an institutional Laffer curve, where institutional intensity first

enhances and then depresses economic performance. North's historical narrative of societies that initially benefit from stronger institutions but later collapse under the weight of bureaucratic expansion finds a precise mathematical expression in this framework.

The policy analysis thus shows that reforms have heterogeneous effects. Verification reforms raise consumption immediately, while supervision and complexity reforms require transitional sacrifices but deliver higher long-run capital and consumption. Welfare comparisons can be reduced to simple shadow-weighted sufficient statistics, consistent with Coase's principle of economizing on costs and Williamson's comparative governance. And institutional intensity may follow a Laffer curve, consistent with North's historical evidence of institutional rise and decline. Together, these results make precise Cheung's claim that transaction costs are institution costs: the entire welfare calculus of institutional reform can be expressed as the present value of resources consumed by institutions and saved by reform.

6. COMPARATIVE GOVERNANCE AND THE BOUNDARY OF THE FIRM

The institutional framework developed above allows us to reinterpret the classical problem posed by Coase (1937) concerning the boundary of the firm, extended by Williamson (1975, 1985) in his comparative governance framework, and given long-run historical significance by North (1990). The basic insight is that markets and hierarchies are not alternative modes chosen by a planner but are instead cost technologies available to private agents. Each governance structure is defined by a triplet of institutional parameters (γ, ϕ, η) , representing supervision, verification, and complexity costs. Choosing between markets, firms, or hybrids is therefore equivalent to choosing the cost vector that minimizes institution costs for the relevant scale and scope of transactions.

Consider two institutional modes, market governance and firm governance, with parameter vectors $(\gamma_M, \phi_M, \eta_M)$ and $(\gamma_F, \phi_F, \eta_F)$ respectively. The steady state under either mode satisfies the Euler condition

$$\alpha A(k^i)^{\alpha-1} = \delta + \rho + \gamma_i + \eta_i k^i,$$

and the resource constraint

$$c^i = \frac{1}{1 + \phi_i} \left(A(k^i)^\alpha - (\delta + \gamma_i)k^i - \frac{1}{2}\eta_i(k^i)^2 \right),$$

where i indexes the chosen governance structure. The welfare associated with each mode can be approximated by the permanent consumption equivalent implied by these steady states. In practice, the decisive factor is the difference in institution costs incurred at the equilibrium scale of activity.

Let k and c represent the relevant scale of capital and consumption. Then the cost differential between firm and market governance is

$$\Delta IC = (\gamma_F - \gamma_M)k^* + (\phi_F - \phi_M)c^* + \frac{1}{2}(\eta_F - \eta_M)(k^*)^2.$$

The efficient governance structure is the one for which this expression is negative, meaning that, at the margin, it absorbs fewer resources from the representative household. This inequality provides a rigorous and quantitative formulation of Coase's original question: firms exist when they reduce the cost of organizing transactions relative to markets. Williamson's discriminating alignment becomes equally precise: when transactions involve high asset specificity and large capital intensity, the terms proportional to k and $(k)^2$ dominate, making firms preferable if they economize on supervision and complexity. When transactions involve frequent, low-specificity exchanges primarily in final consumption goods, the term proportional to c dominates, making markets preferable if verification costs are lower there.

This condition also has dynamic implications. Suppose an economy initially organizes transactions under markets but shifts to firm governance at time zero. Since capital is predetermined, it begins at k^M , the steady state associated with markets, but the new equilibrium under firms, k^F , differs. If $k^F > k^M$, consumption must jump downward immediately to place the economy on the stable path to the new higher steady state, as resources are redirected toward capital accumulation. If $k^F < k^M$, consumption jumps upward, reflecting the liquidation of capital under the new governance structure. Thus, governance choice involves not only differences in long-run institution costs but also short-run sacrifices or windfalls. This formalizes North's observation that institutional transitions are often politically contested because the short-run distributional consequences can outweigh the long-run efficiency gains for some groups.

Uncertainty and asset specificity, emphasized by Williamson (1985), can be modeled as shifts in the institutional parameters themselves. High asset specificity raises the risk of opportunism in market exchange, effectively increasing ϕ_M , while firm governance can mitigate such risks, implying a lower ϕ_F . Conversely, high uncertainty or technological complexity increases the bureaucratic burden of hierarchies, raising η_F relative to η_M . The comparative governance inequality then determines which institutional mode minimizes total costs given the characteristics of the transaction. This formulation unites Williamson's contractual analysis with the dynamic general equilibrium of growth: the representative household bears the institutional costs through its budget constraint, and governance choice determines not only steady-state allocations but also transitional dynamics.

Historical experience illustrates the same principle. In periods of rapid industrialization, when asset specificity and capital intensity are high, firms

expand because hierarchical organization economizes on verification and opportunism, even at the price of higher bureaucratic complexity. In periods of extensive trade in standardized goods, markets dominate because they minimize verification costs relative to the costs of maintaining bureaucracies. North's historical analysis of institutional divergence is captured in the dynamic differences between these modes: societies that persist with high-cost governance structures converge to lower steady states of capital and consumption, while societies that innovate by shifting to lower-cost governance converge to higher steady states.

In this way, the boundary of the firm is not a theoretical abstraction but the solution to a concrete inequality comparing the shadow-valued costs of markets and hierarchies. Coase's insight that firms exist to economize on the cost of using the price mechanism, Williamson's discriminating alignment of governance modes with transactional attributes, North's historical thesis on the role of institutional change in long-run development, and Cheung's equivalence of transaction and institution costs all converge in a single analytic framework. Institutions are technologies of organization, and the choice among them is determined by which minimizes the real, shadow-valued costs borne by the capitalist household.

7. INSTITUTIONAL CHANGE AND LONG-RUN GROWTH

The framework can now be extended to study institutional change and its consequences for long-run growth. In capitalist economies, institutions evolve not as the outcome of a social planner's design, but as the cumulative effect of innovations in contracting, governance, and enforcement mechanisms adopted by households and firms seeking to economize on costs. As Coase (1937, 1960) observed, institutions arise endogenously because the costs of using the market or of managing within a firm vary across contexts. Williamson (1975, 1985) added that comparative governance involves aligning transactional attributes with governance structures that minimize costs. North (1990, 2005) emphasized that the long-run divergence of economies stems from their ability, or inability, to reduce the costs of exchange over time. Cheung (1998) closed the loop by asserting that transaction costs are in fact institution costs—real resources consumed by private agents to make trade feasible.

In our dynamic general equilibrium system, institutional change corresponds to the evolution of the parameters $(\gamma(t), \phi(t), \eta(t))$ over time. These parameters directly enter both the resource constraint and the Euler equation of the capitalist household. The household's accumulation equation,

$$\dot{k}(t) = Ak(t)^\alpha - (\delta + \gamma(t))k(t) - \frac{1}{2}\eta(t)k(t)^2 - (1 + \phi(t))c(t),$$

shows that any shift in institutional costs alters the net savings available to finance capital accumulation. The Euler equation,

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma} \left(\alpha A k(t)^{\alpha-1} - \delta - \rho - \gamma(t) - \eta(t)k(t) \right),$$

demonstrates that supervision and complexity costs reduce the effective return to capital and thereby depress the growth rate of consumption. Institutional change is therefore not a background condition but a direct determinant of both the household's intertemporal choices and the economy's growth trajectory.

Two societies with identical technology, preferences, and initial capital may diverge permanently if their institutional trajectories differ. If one society gradually reduces supervision, verification, and complexity costs—through improved property rights enforcement, contract standardization, or simplification of bureaucratic processes—its representative households face higher effective returns on capital and lower resource drains. Consumption growth accelerates, capital accumulation rises, and the economy converges toward a higher steady state. By contrast, if another society allows supervision costs to remain high, verification costs to proliferate, or bureaucratic complexity to intensify with scale, then capital accumulation is stunted and the growth of consumption is slowed. In dynamic terms, the Euler equation ensures that such a society converges to a lower steady state. This formalizes North's claim that institutional divergence generates economic divergence.

The comparison between the Ramsey steady state and the institutional golden rule underscores this result. The golden rule level of capital satisfies

$$\alpha A (k^{GR})^{\alpha-1} = \delta + \gamma + \eta k^{GR},$$

and represents the capital intensity that maximizes consumption in the long run. Both the Ramsey steady state and the golden rule decline as supervision and complexity costs increase, but because the Ramsey equilibrium also incorporates time preference, the gap between the two widens under poor institutions. This widening efficiency gap corresponds precisely to North's historical argument that inefficient institutions not only depress long-run output but also prevent societies from ever achieving their full potential.

Institutional reform, however, is rarely politically costless. As the dynamics of Sections 5 and 6 have shown, reductions in supervision or complexity costs often require an immediate fall in consumption, since households must save more in order to exploit higher long-run returns. Elites who benefit from rents associated with supervision or bureaucracy resist such reforms, preferring the preservation of inefficient arrangements even

if long-run output is depressed. The political economy of reform therefore reflects the same transitional dynamics as the household's optimization: immediate sacrifices offset by long-run gains. This helps explain why, as North (1990) emphasized, inefficient institutions often persist despite their evident growth costs.

The process of institutional innovation can be conceptualized as movement along an institutional frontier in (γ, ϕ, η) -space. Economies closer to the efficient frontier, where institutional costs are minimized, enjoy higher steady-state capital and consumption and converge more rapidly to equilibrium. Economies further inside the frontier suffer from lower capital accumulation and slower consumption growth. Movement toward the frontier may be achieved through innovations in governance, such as the rise of relational contracting, judicial independence, or streamlined property registration systems. The capitalist household benefits directly from such innovations because they reduce the real resources absorbed by institutions, increasing both feasible consumption and the effective return on capital.

Long-run convergence is therefore conditional not only on technology and preferences but also on institutions. Societies that succeed in lowering institutional costs experience higher growth and converge to higher steady states, while those that fail remain trapped at low levels of capital and consumption. In formal terms, if one society reduces its cost parameters so that $(\gamma_X, \phi_X, \eta_X) < (\gamma_Y, \phi_Y, \eta_Y)$, then its steady-state allocations satisfy $k^X > k^Y$, $c^X > c^Y$, and $W_X > W_Y$. Institutional divergence thus leads to welfare divergence, reinforcing North's contention that the roots of prosperity and poverty lie in institutional evolution.

In summary, institutional change in this model is not an exogenous backdrop but a central determinant of dynamic accumulation paths. By entering both the resource constraint and the Euler equation of the capitalist household, institutions shape not only long-run outcomes but also transitional dynamics and welfare. Coase's idea that institutions economize on the cost of using markets, Williamson's theory that governance must align with transaction attributes, North's historical evidence of divergence, and Cheung's assertion that transaction costs are institution costs all converge in this analysis. Institutional change is the hinge upon which capitalist growth turns: societies that reduce supervision, verification, and complexity costs achieve sustained growth, while those that fail to do so stagnate.

8. CONCLUSION

This paper has developed a dynamic general equilibrium framework that embeds institutions directly into the intertemporal choices of a representative capitalist household. By conceptualizing transaction costs as institution costs, following Cheung (1998), we have shown how governance

structures absorb real resources and alter the incentives facing producers, consumers, and owners of firms. Institutions are not treated as background conditions or as constraints imposed by a benevolent planner, but rather as technologies of organization that the capitalist household must pay for in order to make exchange feasible. In this sense, institutions operate as both drains on resources and wedges in dynamic accumulation, simultaneously reducing feasible consumption and lowering the effective return on capital.

The model unifies the insights of Coase (1937, 1960), Williamson (1975, 1985), North (1990, 2005), and Cheung (1998). Coase argued that firms exist to economize on the cost of using the price system, a principle that here takes the form of a quantitative boundary condition: governance structures are efficient when their cost vector (γ, ϕ, η) minimizes the shadow-valued resources absorbed at the relevant scale of capital and consumption. Williamson's theory of comparative governance is made precise by expressing markets, firms, and hybrids as alternative parameter sets, where high asset specificity or scale favors hierarchies, and frequent standardized transactions favor markets. North's historical thesis that the divergence of societies lies in institutional evolution is captured directly in the dynamics: economies with persistently high supervision, verification, and complexity costs converge to low steady states, while those that reduce such costs converge to higher ones. Cheung's claim that transaction costs are institution costs is given exact operational meaning, since the welfare value of institutional reform is the present value of resources freed for productive use.

The comparative statics of the model show that supervision and complexity costs depress both capital and consumption, while verification costs act as a level wedge on feasible consumption. The Euler equation demonstrates that these costs reduce the growth rate of consumption by lowering effective returns, and the golden rule comparison reveals that poor institutions widen the efficiency gap between potential and realized outcomes. Policy experiments illustrate the different profiles of reform: reductions in verification costs deliver immediate consumption gains, while reductions in supervision and complexity costs require transitional sacrifices but raise long-run capital and consumption. Welfare comparisons can be reduced to shadow-weighted sufficient statistics, allowing institutional reforms to be ranked by their return on investment. The concept of an institutional Laffer curve emerges naturally when governance intensity both improves productivity and consumes resources, producing an interior optimum where marginal benefits equal marginal costs.

The broader implication of this analysis is that institutions are not exogenous constraints but endogenous cost technologies, directly shaping household budgets, accumulation, and welfare. A capitalist economy does not rely on a planner to optimize allocations, but rather on the decentralized

decisions of households who must choose governance structures that minimize institution costs. The model shows precisely how those choices determine long-run growth trajectories and why institutional reforms matter so profoundly for development. By anchoring institutional economics firmly within the dynamic general equilibrium of the Ramsey–Cass–Koopmans model, while respecting Coase’s focus on cost economization, Williamson’s emphasis on comparative governance, North’s concern with historical divergence, and Cheung’s identity between transaction and institution costs, we provide a rigorous synthesis that links institutional theory to modern growth dynamics.

The conclusion is therefore clear. Economic performance in capitalist systems is shaped not only by technology and preferences but also, fundamentally, by the costliness of institutions. Societies that succeed in reducing these costs enjoy higher accumulation, faster growth, and greater welfare, while those that fail remain trapped in low-level equilibria. Institutions are not ancillary but central, not abstract but concrete, and not costless but costly. They are the real technologies of organization through which capitalism operates, and their efficiency or inefficiency determines the fate of long-run growth.

APPENDIX: PROPOSITIONS AND PROOFS

We work with the decentralized institutional Ramsey–Cass–Koopmans (RCK) system on $\Omega = (0, \infty)^2$:

$$\dot{k} = G(k, c) \equiv F(k) - \delta k - c - \Phi(k, c), \quad \dot{c} = H(k, c),$$

with CRRA preferences (elasticity $\sigma > 0$), discount rate $\rho > 0$, depreciation $\delta \geq 0$, production $F \in C^2((0, \infty))$ strictly concave ($F' > 0, F'' < 0$), and institutional costs $\Phi \in C^2$ (specified per proposition). We adopt the admissibility (“small institution”) condition

$$(A) \quad \sup_{k>0} \{F(k) - (\delta + \gamma)k - \tfrac{1}{2} \eta k^2\} > 0$$

whenever $\Phi(k, c)$ contains the separable quadratic terms $\gamma k + \tfrac{1}{2} \eta k^2 + \phi c$. (Economically: net resources at some scale remain strictly positive, excluding degenerate cases in which excessive institutional burdens force negative consumption.)

Below, “steady state” means an interior fixed point $(k, c) \in \Omega$ satisfying $\dot{k} = \dot{c} = 0$. For separable Φ we often write

$$\Phi(k, c) = \psi(k) + \chi(c), \quad \psi'(k) = \Phi_k, \quad \chi'(c) = \Phi_c.$$

PROPOSITION 1 (Uniqueness and Saddle Stability under Separable Convex Costs).

Assumptions. $F(k) = Ak^\alpha$ with $A > 0, 0 < \alpha < 1$. $\Phi(k, c) = \gamma k + \frac{1}{2}\eta k^2 + \phi c$ with $\gamma, \eta, \phi \geq 0$ and (A).

Claim. There exists a unique interior steady state (k^*, c) with $k, c > 0$. The Jacobian J at (k, c) has $\det J < 0$; hence (k, c) is a (hyperbolic) saddle.

Proof. Uniqueness of k^* . Define $h(k) = F'(k) - (\delta + \rho + \gamma + \eta k)$. Since $F'(k) = \alpha Ak^{\alpha-1}$ is strictly decreasing from $+\infty$ to 0 and the linear term is strictly increasing, there exists a unique $k > 0$ with $F'(k) = \delta + \rho + \gamma + \eta k^*$.

Positivity and uniqueness of c^* . From $\dot{c} = 0$,

$$(1 + \phi)c = F(k) - (\delta + \gamma)k - \frac{1}{2}\eta(k)^2.$$

Using $kF'(k) = \alpha F(k)$ (Cobb–Douglas homogeneity) and the Euler condition, this equals

$$(1 - \alpha)F(k) + \rho k + \frac{1}{2}\eta(k)^2 > 0,$$

so $c^* > 0$. Admissibility (A) guarantees uniform positivity and rules out “too large” (γ, ϕ, η) .

Stability. The Jacobian at steady state is

$$J = \begin{bmatrix} \frac{\rho}{\sigma} & -\frac{\rho}{\sigma} \\ \frac{\partial}{\partial k}(F'(k) - \eta k) & 0 \end{bmatrix}.$$

Its determinant is negative because $F''(k) - \eta < 0$, which ensures the existence of a saddle. ■

Thus, with convex separable institutional costs, institutions absorb resources but do not introduce strategic cross-effects. The unique saddle path survives (classical RCK geometry), yet steady-state levels are lower because γ, η, ϕ divert output to governance. The “smallness” condition ensures the economy remains interior (positive c, k).

PROPOSITION 2 (Multiplicity under Non-Monotone Supervisory Costs).

Assumptions. $F(k) = Ak^\alpha$ with $A > 0, 0 < \alpha < 1$. Let

$$\Phi(k, c) = \gamma k + \frac{1}{2}\eta k^2 - \theta k^\beta + \phi c, \quad \gamma, \eta, \phi \geq 0, \theta > 0, 1 < \beta < 2.$$

Claim. There exists $\theta > 0$ such that the Euler condition has three distinct roots $k_L < k_M < k_H$, generating three steady states (k_i, c_i) with $c_i > 0$. At k_L and k_H , $\det J < 0$ (saddles); at k_M , $\det J > 0$ and $\text{tr} J^* = \rho > 0$ (unstable).

Proof. Geometry of the Euler condition. Define

$$E(k; \theta) = F'(k) - (\delta + \rho + \gamma + \eta k - \theta \beta k^{\beta-1}).$$

For $1 < \beta < 2$, the map $k \mapsto \delta + \rho + \gamma + \eta k - \theta \beta k^{\beta-1}$ is U-shaped (its derivative is $\eta + \theta \beta (1 - \beta) k^{\beta-2} > 0$ near 0, decreases, then increases). Since F' is strictly decreasing, sufficiently large θ yields three intersections. Formalizing: a saddle-node bifurcation arises at (k, θ) solving

$$E(k; \theta) = 0, \quad \partial_k E(k; \theta) = 0,$$

with

$$\theta = \frac{F'(k) - (\delta + \rho + \gamma + \eta k)}{\beta(k)^{\beta-1}} = \frac{\eta - F''(k)}{\beta(\beta - 1)(k)^{\beta-2}}.$$

Under $F(k) = Ak^\alpha$, the scalar equation for k has a solution whenever $\beta > 2 - \alpha$ (mild; e.g., $\alpha = 0.35 \Rightarrow \beta > 1.65$). By standard nondegeneracy ($\partial_\theta E > 0, \partial_k^2 E \neq 0$), there are exactly three roots $k_L < k_M < k_H$.

Positivity of c . From $\dot{k} = 0$,

$$(1 + \phi)c = F(k) - (\delta + \gamma)k - \frac{1}{2}\eta k^2 + \theta k^\beta = (1 - \alpha)F(k) + \rho k + \frac{1}{2}\eta k^2 + (1 - \beta)\theta k^\beta,$$

using $kF'(k) = \alpha F(k)$ and the Euler condition. Hence, the RHS is > 0 , so $c > 0$.

Stability classification. For separable institutional costs $\Phi(k, c) = \psi(k) + \phi c$, the Jacobian of the dynamical system at a steady state (k, c) simplifies to,

$$J = \begin{bmatrix} \frac{\rho}{\sigma}(F'''(k) - \psi''(k)) & -(1 - \phi) \\ 0 & 0 \end{bmatrix}.$$

At a downward crossing ($E_k < 0$, the low and high roots), $\det J < 0$: saddles. At the upward crossing ($E_k > 0$, the middle root), $\det J > 0$ with $\text{tr} J = \rho > 0$: unstable (node/focus). ■

Therefore, non-monotone supervisory costs encode “learning-at-low-scale” (marginal institutional cost falls as k rises from very low levels) and “congestion-at-high-scale” (marginal cost rises later). The result is path dependence: a low-capital institutional trap and a high-capital equilibrium may both be feasible; initial conditions (or small shocks) select the long-run outcome.

PROPOSITION 3 (Complex Eigenvalues from Capital–Consumption Cross-Effects).

Assumptions. $F(k) = Ak^\alpha$, $A > 0, 0 < \alpha < 1$.

$$\Phi(k, c) = \gamma k + \frac{1}{2}\eta k^2 + \phi c + \frac{1}{2}\mu c^2 - \xi k c, \quad \gamma, \eta, \phi, \mu \geq 0, \quad \xi > 0.$$

Assume $1 + \Phi_c(k, c) > 0$ and $D(k, c) := 1 + \frac{c}{\sigma} \frac{\Phi_{cc}}{1 + \Phi_c} > 0$ on Ω .

Claim. There is a nonempty open set of parameters for which the unique steady state (k, c) is an unstable focus (a complex conjugate pair with positive real part).

Proof. First derive the Jacobian in general. The Euler FOC and costate yield (algebra standard in continuous-time consumption dynamics)

$$\dot{c} = \frac{\frac{c}{\sigma}(F' - \delta - \rho - \Phi_k) - \frac{c}{\sigma} \frac{\Phi_{ck}}{1 + \Phi_c} G(k, c)}{D(k, c)}, \quad G = F - \delta k - c - \Phi.$$

At a steady state $G = 0$ and $F' - \delta - \rho - \Phi_k = 0$, and differentiation gives

$$J = \begin{bmatrix} \frac{c}{\sigma D} [F''(k) - \Phi_{kk} - \frac{\rho \Phi_{ck}}{(1 + \Phi_c)}] & -(1 + \phi_c) \\ 0 & 0 \end{bmatrix}.$$

For the stated Φ , $\Phi_{kk} = \eta$, $\Phi_{ck} = -\xi$, hence

$$\det J = \frac{c}{\sigma D} [(1 + \phi_c)(F''(k) - \eta) + \rho \xi]$$

Since $\text{tr} J = \rho > 0$, the eigenvalues are complex iff the discriminant $\Delta = \rho^2 - 4 \det J < 0$, i.e.,

$$\det J > \rho^2/4.$$

Choosing ξ large enough (and adjusting μ so $1 + \Phi_c > 0$) makes $\det J > \rho^2/4$. By continuity, this holds on an open parameter set. Hence the eigenvalues form a complex pair with real part $\rho/2 > 0$: an unstable focus. ■

Economic intuition is obvious: The cross-term $-\xi kc$ captures complementarities: governing a larger, more monetized economy can be cheaper per unit when activities co-move. This strengthens feedback in the Euler dynamics and can produce spirals (oscillations) rather than monotone convergence—formalizing institutional boom–bust episodes.

PROPOSITION 4 (No Cycles under Separable Convex Costs). *Assumptions.* $F \in C^2$ with $F'' < 0$. $\Phi(k, c) = \psi(k) + \chi(c)$ with $\psi'', \chi'' \geq 0$ and $1 + \chi'(c) > 0$.

Claim. The planar system on Ω admits no nontrivial periodic orbits.

Proof. (Hirsch–Smith).

Compute the off-diagonal partials:

$$\frac{\partial \dot{k}}{\partial c} = -(1 + \chi'(c)) < 0, \quad \frac{\partial \dot{c}}{\partial k} = \frac{c}{\sigma D(k, c)} (F''(k) - \psi''(k)) < 0,$$

since $F'' < 0, \psi'' \geq 0, c > 0, D > 0$. Thus the system is strongly competitive on Ω . By the Hirsch–Smith theorem for planar competitive systems, periodic orbits cannot exist; every compact limit set is an equilibrium. ■

Hence, with separable convex costs, governance adds friction but no strategic interaction across margins. The induced order structure precludes rotation in phase space: oscillations are globally ruled out, and the economy converges monotonically to its unique saddle path.

PROPOSITION 5 (Limit Cycles under Cross-Effects or Nonconvexity). *Assumptions. $F'' < 0$. $\Phi \in C^2$ with either cross-effects $\Phi_{ck} \neq 0$ or a nonconvex component causing an unstable focus at (k, c) . Assume*

(i) $1 + \Phi_c \geq q_0 > 0, D \geq d_0 > 0$ on Ω ;
(ii) *coercivity at large scale: for some positive-definite quadratic form Q and $M > 0$,*

$\Phi(k, c) \geq Q(k, c) - M$ *when $\sqrt{k^2 + c^2}$ is large;*

(iii) *no other equilibria in an annulus around (k, c) .*

Claim. There exists at least one periodic orbit surrounding (k, c) .

Proof. Inner boundary (outward flow). Since (k, c) is a hyperbolic repelling focus ($\Delta < 0, \text{tr} J > 0$), the linearized system has a positive-definite Lyapunov function $V_{\text{in}}(x) = x^\top P x$ with $\dot{V}(x) \geq \lambda \|x\|^2 > 0$ near the origin (standard Lyapunov equation). Thus a small closed curve Γ_{in} around (k, c) is crossed outward.

Outer boundary (inward flow). Let $V_{\text{out}}(k, c) = k^2 + c^2$. For large radius R , coercivity yields

$$G(k, c) = F(k) - \delta k - c - \Phi(k, c) \leq -Q(k, c) + C_1 R + C_2 \leq -\frac{\lambda_Q}{2} R^2$$

for some $\lambda_Q > 0$. Using the explicit $\dot{c}$ formula with bounded Φ_{ck} , $1 + \Phi_c \geq q_0$ and $D \geq d_0$, one obtains $\dot{V}_{\text{out}} = 2k\dot{k} + 2c\dot{c} \leq -CR^3$ for large R : the vector field points inward on the circle $\Gamma_{\text{out}} = \{k^2 + c^2 = R^2\}$.

Trapping annulus and Poincaré–Bendixson. The closed annulus $\mathcal{A}$ between Γ_{in} and Γ_{out} is positively invariant. By (iii), $\mathcal{A}$ contains no equilibria other than the inner repeller. Therefore by the Poincaré–Bendixson theorem, the ω -limit set of any trajectory in $\mathcal{A} \setminus \{(k, c)\}$ is a periodic orbit. ■

Economic discussion. Cross-effects/nonconvexities create a repelling focus (inner source). Coercive institutional burdens at scale push trajectories back (outer sink). The economy cannot settle at the repeller nor escape to infinity; instead it cycles endogenously—institutional boom–bust dynamics in which accumulation relaxes costs for a while, then escalating complexity raises costs and reverses course.

Notes on Admissibility and Interior Dynamics

- The “small-institution” condition (A) is economically essential: if γ, η, ϕ are too large, the numerator in $(1 + \phi)c = F(k) - (\delta + \gamma)k^* - \frac{1}{2}\eta k^2$ becomes nonpositive, forcing $c \leq 0$. All results above are stated under (A) (and,

where relevant, $1 + \Phi_c > 0, D > 0$), ensuring interior steady states and economically meaningful paths.

- For Proposition 2, multiplicity requires the U-shape in the marginal institutional cost, which holds for $1 < \beta < 2$ (the $\beta \in (0, 1)$ case cannot generate three intersections because $k^{\beta-1}$ is decreasing too steeply and does not bend the user-cost upward at high k).
- For Proposition 3, complex eigenvalues follow from $\det J > \rho^2/4$. The cross-effect $\Phi_{ck} = -\xi < 0$ enters positively in $\det J$ via $+\rho\xi/(1+\Phi_c^*)$, giving a clean, checkable inequality.
- Propositions 4–5 describe a sharp dichotomy: separable convex costs $\Rightarrow$ global monotonicity (no cycles); cross-effects/nonconvexity $\Rightarrow$ possibility of cycles via Poincaré–Bendixson.

References to Mathematical Results Used

- Hirsch, M. W. (1982). “Systems of differential equations that are competitive or cooperative: Limit sets.” *SIAM J. Math. Anal.*, 13(2), 167–179.
- Smith, H. L. (1995). *Monotone Dynamical Systems: An Introduction to the Theory of Competitive and Cooperative Systems*. AMS.
- Perko, L. (2001). *Differential Equations and Dynamical Systems* (3rd/4th ed.). Springer. (Poincaré–Bendixson, Lyapunov constructions).

REFERENCES

- Acemoglu, Daron, Simon Johnson, and James A. Robinson, 2001. The Colonial Origins of Comparative Development: An Empirical Investigation. *American Economic Review* **91**(5), 1369–1401.
- Acemoglu, Daron, and James A. Robinson, 2012. *Why Nations Fail: The Origins of Power, Prosperity, and Poverty*. New York: Crown Business.
- Alchian, Armen A., and Harold Demsetz, 1972. Production, Information Costs, and Economic Organization. *American Economic Review* **62**(5), 777–795.
- Barzel, Yoram, 1997. *Economic Analysis of Property Rights*. 2nd ed. Cambridge: Cambridge University Press.
- Cheung, Steven N. S., 1998. The Transaction Costs Paradigm. *Economic Inquiry* **36**(4), 514–521.
- Coase, Ronald H., 1937. The Nature of the Firm. *Economica* **4**(16), 386–405.
- Coase, Ronald H., 1960. The Problem of Social Cost. *Journal of Law and Economics* **3**(1), 1–44.
- Coase, Ronald H., 1992. The Institutional Structure of Production. *American Economic Review* **82**(4), 713–719. (Nobel Prize Lecture, December 9, 1991).
- Demsetz, Harold, 1967. Toward a Theory of Property Rights. *American Economic Review* **57**(2), 347–359.
- Greif, Avner, 2006. *Institutions and the Path to the Modern Economy: Lessons from Medieval Trade*. Cambridge: Cambridge University Press.

- Nelson, Richard R., and Sidney G. Winter, 1982. *An Evolutionary Theory of Economic Change*. Cambridge, MA: Harvard University Press.
- North, Douglass C., 1990. *Institutions, Institutional Change, and Economic Performance*. Cambridge: Cambridge University Press.
- North, Douglass C., 2005. *Understanding the Process of Economic Change*. Princeton: Princeton University Press.
- North, Douglass C., and Robert P. Thomas, 1973. *The Rise of the Western World: A New Economic History*. Cambridge: Cambridge University Press.
- North, Douglass C., and Barry R. Weingast, 1989. Constitutions and Commitment: The Evolution of Institutions Governing Public Choice in Seventeenth-Century England. *Journal of Economic History* **49**(4), 803–832.
- Ostrom, Elinor, 1990. *Governing the Commons: The Evolution of Institutions for Collective Action*. Cambridge: Cambridge University Press.
- Williamson, Oliver E., 1975. *Markets and Hierarchies: Analysis and Antitrust Implications*. New York: Free Press.
- Williamson, Oliver E., 1985. *The Economic Institutions of Capitalism: Firms, Markets, Relational Contracting*. New York: Free Press.
- Williamson, Oliver E., 2010. Transaction Cost Economics: The Natural Progression. *American Economic Review* **100**(3) 673–690. (Nobel Prize Lecture, December 8, 2009).